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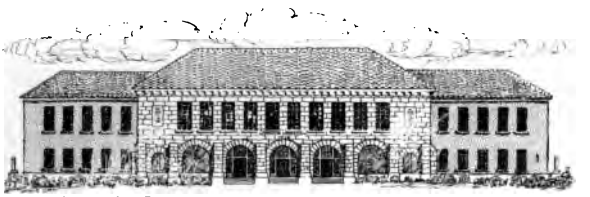

NEW

Elementary Arithmetic,

EMBRACING

MENTAL AND WRITTEN EXERCISES,

FOR BEGINNERS.

PREPARED TO ACCOMPANY THE MATHEMATICAL SERIES OF

BENJAMIN GREENLEAF, A. M.

BY THE EDITOR OF "NEW PRACTICAL ARITHMETIC," "NEW ELEMENTARY ALGEBRA," ETC., IN THE SERIES.

LEACH, SHEWELL, AND SANBORN,
BOSTON AND NEW YORK.

GREENLEAF'S

COMPREHENSIVE SERIES.

NEW PRIMARY ARITHMETIC.

NEW ELEMENTARY ARITHMETIC.

NEW PRACTICAL ARITHMETIC.

MANUAL OF INTELLECTUAL ARITHMETIC.

PREFACE.

IN mental culture, no less than in manual labor, it is desirable to have instruments especially adapted to the work to be performed. Of late it has become very evident that there are numerous learners, of limited opportunities, who require an easy, comprehensive course of arithmetic. Such, having no time to master the several books of a consecutive series, however excellent, demand, in a single book, the more important principles of the science, combined with the more useful applications to ordinary business computations.

In most Graded Schools, also, there may be found a class of pupils who might advantageously use a brief elementary course, uniting mental and written exercises, before passing to any more elaborate treatise. Experience has demonstrated that beginners are best initiated into any branch by means of repeated simple illustrations and numerous easy examples.

These conditions have been carefully kept in view, during the preparation of this work, and no labor has been spared to embody such improved methods of putting things, sanctioned by the best educators, as may not heretofore have found a place in any text-book.

The work is analytic and inductive. Mental and written exercises have been combined, with the same forms of analysis applied to both. By this plan, not only a volume, of separate mental arithmetic is rendered superfluous, but valuable time is saved, and the pupil's progress promoted by an order at once natural and scientific. The number of topics treated is necessarily limited, yet will be found sufficient in range to prepare for common business.

The great favor with which the Elementary Arithmetic has been everywhere received, has induced the editor to take advantage of the necessity of the renewal of the electrotype plates, to make it still more attractive and useful.

Questions which may be omitted at the option of the teacher have been denoted by °.

Bank Discount, and United States Bonds, and the Metric System have been included among the topics treated; copious Dictation Exercises, mental and written, and Test Examples have been appended for the convenience of teachers, and to furnish additional practice for the pupil.

This work, and the Primary which precedes, it is believed, form a very simple and practical SHORTER COURSE IN ARITHMETIC, especially adapted to the wants of many learners.

SILVER LAKE, MASS., *May*, 1882.

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NEW
ELEMENTARY ARITHMETIC.

NUMBERS.

1.	A **Unit** is a single thing of any kind, or	**one.**
2.	One unit and one unit are two units, or	**two.**
3.	Two units and one unit are three units, or	**three.**
4.	Three units and one unit are four units, or	**four.**
5.	Four units and one unit are five units, or	**five.**
6.	Five units and one unit are six units, or	**six.**
7.	Six units and one unit are seven units, or	**seven.**
8.	Seven units and one unit are eight units, or	**eight.**
9.	Eight units and one unit are nine units, or	**nine.**

What is a Unit? What are one and one called? Two and one? Three and one? Four and one? Five and one? Six and one? Seven and one? Eight and one?

10. Quantity is anything that can be measured, or computed. Thus,

Distance is quantity, since it can be measured, so as to be named miles, rods, etc.

11. Numbers express how many units there are in any given quantity. NUMBERS are, therefore, one or more units. Thus,

One, two, three, four, five, six, etc., are numbers.

12. Like Numbers are such as express the same kind of quantity. Thus

Five *dollars* and seven *dollars* are like numbers.

13. An **Operation,** or **Process,** is the work done with numbers.

14. A **Rule** is the direction for performing an operation, or process.

A **Principle** is a general truth.

15. Arithmetic treats of numbers, and is the art of reckoning by them.

FIGURES.

16. Figures are certain marks, or characters, used to express numbers.

Ten different figures are used in expressing numbers.

Names, or value denoted.	*Figures as printed.*	*Figures as written.*	*Names, or value denoted.*	*Figures as printed.*	*Figures as written.*
Cipher,	0	*0*	Five,	5	*5*
One,	1	*1*	Six,	6	*6*
Two,	2	*2*	Seven,	7	*7*
Three,	3	*3*	Eight,	8	*8*
Four,	4	*4*	Nine,	9	*9*

What is Quantity? Why is distance quantity? What do Numbers express? Mention some numbers. What are Like Numbers? What is an Operation? A Rule? Arithmetic? What are Figures?

17. The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, are called **Significant Figures**, or **Numerals**, because each *signifies*, or stands for, as many units as its name denotes.

18. The figure 0, or cipher, is sometimes called **Zero**, or **Naught**, because, when used alone, it stands for *no number*. Thus,

0 dollars means *no* dollars.

EXERCISES.

Write the following figures and name each of them: —

(1)	(2)	(3)	(4)	(5)	(6)
1	*8*	*0*	*7*	*2*	*9*
2	*9*	*1*	*2*	*1*	*7*
3	*6*	*4*	*8*	*3*	*8*
4	*3*	*6*	*8*	*4*	*6*
6	*2*	*9*	*5*	*9*	*0*
7	*4*	*3*	*4*	*7*	*5*
0	*5*	*2*	*1*	*6*	*4*

Write in figures arranged in columns: —

7. One, three, four, two, seven, five, six, eight.
8. Four, one, five, nine, eight, two, six.
9. Five, nine, two, eight, six, four, three, seven.
10. Six, cipher, three, seven, two, five, one, four.
11. Seven, nine, three, zero, two.
12. Nine, one, zero, five, seven, eight, four.

Which figures are called Significant Figures? Why? Which is called Zero or Naught? Why?

UNITS, TENS.

19. Nine units and one unit are	**ten.**
One ten and one unit are	**eleven.**
One ten and two units are	**twelve.**
One ten and three units are	**thirteen.**
One ten and four units are	**fourteen.**
One ten and five units are	**fifteen.**
One ten and six units are	**sixteen.**
One ten and seven units are	**seventeen.**
One ten and eight units are	**eighteen.**
One ten and nine units are	**nineteen.**

Thus, "teen" means "and ten."

20. Two tens are	**twenty.**
Three tens are	**thirty.**
Four tens are	**forty.**
Five tens are	**fifty.**
Six tens are	**sixty.**
Seven tens are	**seventy.**
Eight tens are	**eighty.**
Nine tens are	**ninety.**

Thus, "ty" means "tens."

21. Exact Tens, as ten, twenty, thirty, etc., are expressed by combining 1, 2, 3, etc., denoting the number of tens, with 0; and the number between exact tens, by combining 1, 2, 3, etc., with a significant figure.

What is an exercise? *Ans.* A lesson for practice. What are nine and one called? What are ten and one called? Ten and two? Ten and three? Ten and four? Ten and five? Ten and six? Ten and seven? Ten and eight? Ten and nine? What is the meaning of "teen"? What are two tens called? Three tens? Four tens? Five tens? Six tens? Seven tens? Eight tens? Nine tens? What is the meaning of "ty"? How are exact tens *expressed*? How are the numbers between exact tens expressed?

22. Numbers expressed by two figures are written, —

Ten,	*10*	Thirty-one,	*31*	Fifty-two,	*52*
Eleven,	*11*	Thirty-two,	*32*	Fifty-three,	*53*
Twelve,	*12*	Thirty-three,	*33*	Fifty-four,	*54*
Thirteen,	*13*	Thirty-four,	*34*	Fifty-five,	*55*
Fourteen,	*14*	Thirty-five,	*35*	Fifty-six,	*56*
Fifteen,	*15*	Thirty-six,	*36*	Fifty-seven,	*57*
Sixteen,	*16*	Thirty-seven,	*37*	Fifty-eight,	*58*
Seventeen,	*17*	Thirty-eight,	*38*	Fifty-nine,	*59*
Eighteen,	*18*	Thirty-nine,	*39*	Sixty,	*60*
Nineteen,	*19*	Forty,	*40*	Sixty-one,	*61*
Twenty,	*20*	Forty-one,	*41*	Sixty-two,	*62*
Twenty-one,	*21*	Forty-two,	*42*	Sixty-three,	*63*
Twenty-two,	*22*	Forty-three,	*43*	Sixty-four,	*64*
Twenty-three,	*23*	Forty-four,	*44*	Sixty-five,	*65*
Twenty-four,	*24*	Forty-five,	*45*	Sixty-six,	*66*
Twenty-five,	*25*	Forty-six,	*46*	Sixty-seven,	*67*
Twenty-six,	*26*	Forty-seven,	*47*	Sixty-eight,	*68*
Twenty-seven,	*27*	Forty-eight,	*48*	Sixty-nine,	*69*
Twenty-eight,	*28*	Forty-nine,	*49*	Seventy,	*70*
Twenty-nine	*29*	Fifty,	*50*	Eighty,	*80*
Thirty,	*30*	Fifty-one,	*51*	Ninety,	*90*

With what figures is thirty-two written? Forty-seven? Fifty? *Sixty-three?*

EXERCISES.

Write the following figures, and read them: —

(1)	(2)	(3)	(4)	(5)
10	*15*	*11*	*19*	*91*
12	*23*	*26*	*55*	*83*
14	*24*	*28*	*36*	*75*
15	*25*	*27*	*57*	*96*
17	*29*	*22*	*38*	*98*
18	*30*	*31*	*50*	*44*
20	*41*	*22*	*61*	*89*
13	*39*	*33*	*65*	*78*
21	*40*	*34*	*70*	*99*

Write in figures arranged in columns: —

6. Ten, sixteen, fifty-two, eighty-eight, nineteen, seventy-three, ninety-seven, thirty-seven.

7. Thirteen, seventeen, forty-five, sixty-two, twenty-seven, forty-two, seventy-one, eighty.

8. Twenty-six, sixty-two, eighteen, eighty-one, seventy-nine, ninety-seven, fifty, five.

9. Nine, ninety, ninety-nine, thirty-five, fifty-three, eighty-two, seventy-two, twenty-two.

10. Thirty-three, forty, fifty-five, sixty-two, eighty-eight, nineteen, ninety-one.

What is the largest number that can be expressed by one figure? By two figures? With what figures do you write eighty? Fifty-*five?*

UNITS, TENS, HUNDREDS.

23. When a number is expressed by two or more figures, written together side by side, the **Value** denoted by a significant figure is according to the *order* in which it is written. Thus,

In 25, the 5, which is in the *first order* from the right, expresses 5 *ones*, or five; and the 2, which is in the *second order*, expresses 2 *tens*, or twenty.

24. Primary Units, or those from which are formed units of the higher orders, are expressed by a figure standing alone, or in the first order at the left of a dot (.), known as the **Decimal Point.** Thus,

By taking together ten units of the first order, and considering the group as a single thing, is formed a unit of the second order, called **One Ten.**

The decimal point, when not written, is understood.

5, or 5., expresses five.

25. Ten tens are called **One Hundred,** which forms a unit of the *third order*, and is written 100

Exact hundreds are, therefore, expressed by writing the figure denoting their number in the third order, with two ciphers at the right. Thus, we write,

Two hundred,	200	Six hundred,	600
Three hundred,	300	Seven hundred,	700
Four hundred,	400	Eight hundred,	800
Five hundred,	500	Nine hundred,	900

According to what is the value expressed by a figure written side by side with others? What is the value of the 5 in 25? Of the 2? What are the units expressed by a figure standing alone called? Why so called? What are ten tens called? One hundred is a unit of what order? How are exact hundreds expressed? In writing a number, where do we place the figure denoting hundreds?

26. In **Writing a Number** expressing units, tens, and hundreds, we place the figure denoting the *hundreds* in the *third*, the figure denoting the *tens* in the *second*, and the figure denoting the *units* in the *first order*, at the left of the decimal point. Thus,

Two hundred sixty-three, or 2 hundreds, 6 tens, and 3 units, is written 263.

Five hundred seven, or 5 hundreds, 0 tens, and 7 units, is written 507.

EXERCISES.

Write in figures arranged in columns: —

1. The numbers between one hundred and one hundred ten.
2. The numbers between one hundred nine and one hundred nineteen.
3. The numbers between one hundred ninety and two hundred.
4. Three hundred; three hundred three; three hundred thirty; three hundred thirty-three.
5. Five hundred nine; nine hundred five; five hundred ninety; nine hundred fifty-nine.
6. Seven hundred seventy-seven; two hundred twenty-two; eight hundred eighty-eight; six hundred six.
7. Two hundred two; five hundred ninety-nine; four hundred seventy-one; nine hundred ninety-nine.
8. Six hundred one; eight hundred thirty-three; four hundred forty-four.

REVIEW. — What is a Unit? (1.) What is Quantity? (10.) Number? (11.) Arithmetic? (15.) Figures? (16.) What is *the meaning of "teen"?* (19.) Of "ty"? (20.)

Write the following figures, and read them: —

(8)	(9)	(10)	(11)	(12)
112	*712*	*443*	*101*	*199*
344	*341*	*312*	*397*	*786*
106	*560*	*707*	*635*	*652*
780	*818*	*900*	*563*	*496*
920	*292*	*601*	*919*	*999*

THOUSANDS.

27. Ten hundreds are **One Thousand,** which forms a unit of the *fourth* order, and is written 1000

Exact Thousands are, therefore, expressed by writing the figure denoting their number in the fourth order, with three ciphers at the right. Thus, we write,

Two thousand,	2000	Six thousand,	6000
Three thousand,	3000	Seven thousand,	7000
Four thousand,	4000	Eight thousand,	8000
Five thousand,	5000	Nine thousand,	9000

28. Ten thousands are **One Ten-thousand,** which forms a unit of the *fifth* order, and is written 10,000;

Also, two ten-thousands, or twenty thousand, 20,000;

Three ten-thousands, or thirty thousand, 30,000:

and so on.

29. Ten ten-thousands are **One Hundred-thousand,** which forms a unit of the *sixth* order, and is written 100,000:

Also, two hundred thousand, 200,000;

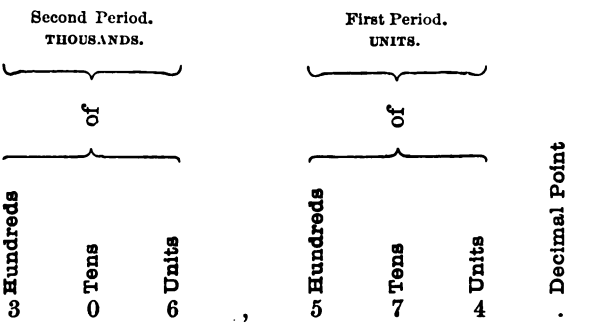
Three hundred thousand, 300,000;

and so on.

What are ten hundreds called? What unit does one thousand form? How are exact thousands written? What are ten thousands called? What unit do ten thousands form? One hundred thousand forms a unit of what order?

30. The first SIX ORDERS OF UNITS, beginning with the primary units, are named: UNITS, *tens*, *hundreds*, THOUSANDS, *ten-thousands*, *hundred-thousands*.

For convenience, in reading numbers expressed by figures, *places*, or *orders of figures*, are separated by a comma (,) into groups, of three figures each, called **Periods.** Each period takes its name from its right-hand order, and has units, tens and hundreds of that name. Thus we have,



where the figures express 3 hundred-thousands 0 ten-thousands 6 *thousands* 5 hundreds 7 tens 4 *units*, or three hundred six thousand five hundred seventy-four.

31. In general, in **Writing a Number** by figures, we write each order of figures expressing its orders of units, and note the absence of any order of units, by a cipher. Thus,

Five thousand twenty, or 5 thousand 0 hundreds 2 tens 0 units, is written 5,020.

One hundred-thousand forms a unit of what order? Give the names of the first six orders. How are figures separated for convenience *in reading*? From what order does each period take its *name*? *Name the* first two periods. In general, how do we write *figures in expressing* numbers?

Fifty thousand two hundred two, or 5 ten-thousands 2 hundreds 2 units, is written 50,202.

Five hundred fifty-five thousand two hundred twenty-two, or 5 hundred-thousands 5 ten-thousands 5 thousands 2 hundreds 2 tens 2 units, is written 555,222.

EXERCISES.

Write in figures and read: —

1. Six units of the fifth order, with seven units of the third order.

Ans. 60,700; read, sixty thousand seven hundred.

2. 3 hundred-thousands 4 ten-thousands 4 thousands.

Ans. 344,000; read, three hundred forty-four thousand.

3. Two units of the fourth order, with three units of the second order.

4. Eight units of the fifth order, with three units of each of the lower orders.

5. 9 units of the sixth order, with no units of the fifth order, and six units of each of the lower orders.

6. 316 in the second period, 0 in each order of the first period.

7. 21 thousands 2 tens and 1 unit.

8. 2 hundred-thousands 5 ten-thousands with five units of the third order.

9. The greatest number that can be expressed by three figures.

10. The greatest number that can be expressed by six figures.

REVIEW. -- What are ten tens called? (25.) Ten hundreds? (27.) Ten thousands? (28.) Ten ten-thousands? (29.) Name the first six orders. (30.) The first two periods. (30.)

NOTATION.

32. Notation is the method of writing numbers.

The method of expressing numbers by figures is called the Arabic, because it was used by the Arabs.

33. The **Periods** are named from the right; thus, *Units*, *thousands*, *millions*, *billions*, *trillions*, etc.

The **Orders** are named from the right; thus,

UNITS, *tens*, *hundreds;* THOUSANDS, *ten-thousands*, *hundred-thousands;* MILLIONS, *ten-millions*, *hundred-millions;* BILLIONS, *ten-billions*, *hundred-billions;* TRILLIONS, *ten-trillions*, *hundred-trillions*, etc. Hence the following

Table.

5th Period. TRILLIONS.			4th Period. BILLIONS.			3d Period. MILLIONS.			2d Period. THOUSANDS.			1st Period. UNITS.		
of			of			of			of			of		
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
5	6	2,	7	9	3,	1	5	4,	2	0	5,	6	3	8.

where the figures express *five hundred sixty-two* TRILLION, *seven hundred ninety-three* BILLION, *one hundred fifty-four* MILLION, *two hundred five* THOUSAND, *six hundred and thirty-eight.*

What is Notation? What is the method of expressing numbers *by figures* called? Why so called? Name the Periods from the *right. Name the Orders.* State the Orders in the table.

The periods above TRILLIONS, in their order, are QUADRILLIONS, QUINTILLIONS, SEXTILLIONS, SEPTILLIONS, etc.

PRINCIPLES OF NOTATION.

34. 1. *Ten units of any order are always equal to one of the next higher.*

For ten units **are one ten, ten tens are** one hundred, ten hundreds **are** one thousand, and so on. Hence,

2. *Each removal of a figure an order towards the left makes the value expressed ten-fold.*

For the value of the units expressed by a figure is determined by the order occupied by the figure.

WRITTEN EXERCISES.

35. 1. Write in figures, seven million six hundred ninety-three thousand two hundred four.

Writing the 7 millions as the only order of the third period, 693 thousands as the orders of the second period, and 204 as the orders of the first period, we have 7,693,204. Hence, the following

Rule. — *Beginning with the highest period to be expressed, write the figures belonging to each period, in their orders, and observe to mark the omission of any order of units with a cipher.*

What periods are above Trillions? How many units of any order are always equal to one of the next higher? How is the value expressed increased by each removal of a figure toward the left? °Recite the Rule for Notation.

Examples.

Write in figures the following numbers: —

2. Seven hundred seventy.
3. One thousand eight hundred eighty-five.
4. Three thousand five hundred fifty-three.
5. Eleven thousand one.
6. Eleven hundred eleven.
7. Seventy-three thousand five hundred ninety-two.
8. Eighty-four thousand nine hundred nine.
9. Two hundred thirty thousand five hundred six.
10. Forty-one thousand nineteen.
11. Nine thousand nine hundred seven.
12. Eighty-nine thousand ninety-seven.
13. Twenty-one thousand one hundred twenty-one.
14. Three hundred thousand six.
15. Fifteen thousand one hundred fifteen.
16. Seventy-nine thousand nine hundred seven.
17. Sixty-seven thousand three hundred six.
18. Six hundred thirty-five thousand four hundred thirty-eight.
19. Forty-two thousand four hundred forty-four.
20. Ninety-eight thousand six hundred nine.
21. Nineteen thousand three hundred fifty-one.
22. One hundred thousand forty-seven.
23. One million ten thousand ten.
24. Sixty-one million sixteen thousand six hundred five.
25. Eight hundred twelve thousand three hundred forty-seven.

What is an Example? *Ans.* A question, or something to be *done, under a rule.*

26. Twelve million twenty thousand three hundred one.

27. Seven million nine hundred twenty-three thousand four hundred six.

28. Three billion one hundred eleven million two hundred twenty thousand two.

29. Five hundred eighty-one million thirty-six thousand twenty-nine.

30. One trillion one million one thousand ninety-one.

31. Twenty-nine million fifty thousand one hundred fifty.

32. One hundred million one hundred thousand one hundred one.

33. Six hundred thirty-one million one hundred twenty-four thousand sixty-six.

34. Five trillion five thousand five.

35. Two hundred ninety trillion six hundred thirty billion four hundred two million four hundred seventy-nine thousand eight hundred fifteen.

NUMERATION.

36. **Numeration** is the method of reading numbers.

WRITTEN EXERCISES.

37. Let it be required to read 7693254.

Pointing off the given figures into periods, from the right, we have 7,693,254.

REVIEW. — What is an Operation? (13.) What is a Rule? (14.) What is the cipher sometimes called? (18.) What do ciphers written with other figures denote, as in 108, 240, etc.? Ans. Vacant orders, or such as have an omission of units.

The third period is 7 *millions*, the second period is 693 *thousands*, the first period is 254 *units*; therefore, the whole reads, seven million six hundred ninety-three thousand two hundred fifty-four. Hence, the following

Rule.—*Beginning at the units' order, point off the given expression into as many periods as possible, of three figures each.*

Then, begin at the left and read each period, giving after each, excepting the last, the name of the period.

Examples.

Point off and read the following:—

2.	932	11.	9907	20.	3992
3.	1865	12.	73421	21.	17700
4.	7060	13.	1306781	22.	100047
5.	11001	14.	300006	23.	13412
6.	1199	15.	21034	24.	712345
7.	73592	16.	71290	25.	814347
8.	170643	17.	635438	26.	91628
9.	230506	18.	2923406	27.	12020301
10.	31031	19.	22443	28.	16312041

29.	631245687	32.	78900672164
30.	123000000000	33.	130132456987
31.	444555333777	34.	29630402479815

35. 100006005700800006000

What is Numeration? At which hand do you begin to point off numbers? At which to read? Recite the Rule. Why is the name of the units' period not given in reading figures? *Ans.* Because the name is understood.

ADDITION.

38. 1. If James has 4 books and John 2, how many books have both of them?

2. Four trees and 3 trees are how many trees? How many are 4 and 3?

3. If a house has 5 windows in the front, and in the side 6, how many windows has it both in the front and in the side?

How many are 5 and 6?

39. **Addition** is the process of finding a number equal to two or more given numbers of the same kind.

The SUM is the number which is equal to the numbers added. Thus,

The sum of two cents and 3 cents is 5 cents.

40. A **Sign** is a mark used to shorten an expression, or to indicate a process.

What is Addition? The Sum or Amount? What is a Solution? *Ans.* The answering of a question, or giving in order the steps required to obtain its answer.

The **Sign of Addition** is an erect cross, +, and is called *plus*. Thus,

3 + 2, which is read three plus two, denotes that 3 and 2 are to be added.

The **Sign of Equality** is two short parallel horizontal lines, and is read *equals*, or *equal to*. Thus,

3 + 2 = 5, is read, three plus two are equal to five.

Addition Tables.

1 and	2 and	3 and	4 and	5 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 3	2 " 4	2 " 5	2 " 6	2 " 7
3 " 4	3 " 5	3 " 6	3 " 7	3 " 8
4 " 5	4 " 6	4 " 7	4 " 8	4 " 9
5 " 6	5 " 7	5 " 8	5 " 9	5 " 10
6 " 7	6 " 8	6 " 9	6 " 10	6 " 11
7 " 8	7 " 9	7 " 10	7 " 11	7 " 12
8 " 9	8 " 10	8 " 11	8 " 12	8 " 13
9 " 10	9 " 11	9 " 12	9 " 13	9 " 14
10 " 11	10 " 12	10 " 13	10 " 14	10 " 15

6 and	7 and	8 and	9 and	10 and
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11
2 " 8	2 " 9	2 " 10	2 " 11	2 " 12
3 " 9	3 " 10	3 " 11	3 " 12	3 " 13
4 " 10	4 " 11	4 " 12	4 " 13	4 " 14
5 " 11	5 " 12	5 " 13	5 " 14	5 " 15
6 " 12	6 " 13	6 " 14	6 " 15	6 " 16
7 " 13	7 " 14	7 " 15	7 " 16	7 " 17
8 " 14	8 " 15	8 " 16	8 " 17	8 " 18
9 " 15	9 " 16	9 " 17	9 " 18	9 " 19
10 " 16	10 " 17	10 " 18	10 " 19	10 " 20

Recite the column 1 and 1. 2 and 1. 3 and 1. 4 and 1. 5 and 1. 6 and 1. 7 and 1. 8 and 1. 9 and 1. 10 and 1. What is a Sign? What is the Sign of Addition? Of Equality?

EXERCISES ON THE TABLES.

1. 2 and 4 are how many?
2. 5 and 1 are how many?
3. 4 and 3 are how many?
4. 3 and 7 are how many?
5. 8 and 2 are how many?
6. 6 and 5 are how many?
7. 9 and 6 are how many?
8. 3 and 8 are how many?
9. 3 and 5 are how many?
10. 7 and 4 are how many?
11. 6 and 6 are how many?
12. 8 and 5 are how many?
13. 4 and 9 are how many?
14. 9 and 2 are how many?
15. 5 and 7 are how many?
16. 8 and 8 are how many?

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17. 2 + 6 are how many?
18. 7 + 7 are how many?
19. 3 + 3 are how many?
20. 5 + 9 are how many?
21. 6 + 7 are how many?
22. 8 + 1 are how many?
23. 4 + 4 are how many?
24. 1 + 9 are how many?
25. 5 + 2 are how many?
26. 7 + 9 are how many?
27. 6 + 3 are how many?
28. 4 + 6 are how many?
29. 8 + 6 are how many?
30. 9 + 9 are how many?

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31. 3 + 9 = how many?
32. 4 + 5 = how many?
33. 10 + 3 = how many?
34. 5 + 5 = how many?
35. 8 + 10 = how many?
36. 7 + 10 = how many?
37. 7 + 1 = how many?
38. 8 + 4 = how many?
39. 1 + 10 = how many?
40. 10 + 6 = how many?
41. 9 + 10 = how many?
42. 10 + 10 = how many?

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43. How many are 13 and 4? 23 and 4? 63 and 4? 93 and 4? 15 and 6? 35 and 6? 6 and 55?

REVIEW. — What is Notation? (32.) Numeration? (36.) What is the Rule for writing numbers? (35.) For reading numbers? (37.)

44. How many are 11 and 7? 21 and 7? 51 and 7? 71 and 7? 12 and 3? 32 and 3? 83 and 3?

45. How many are 16 and 2? 46 and 2? 46 and 5? 66 and 5? 96 and 5? 17 and 3? 37 and 3?

46. How many are 18 and 7? 28 and 7? 68 and 7? 98 and 7? 14 and 7? 27 and 7?

47. How many are 19 and 8? 29 and 8? 69 and 8? 89 and 9? 10 and 9? 20 and 9? 80 and 9?

MENTAL EXERCISES.

1. John has 4 apples, Arthur 3, and Henry 2; how many have they in all?

SOLUTION. — *They have the sum of* 4 *apples*, 3 *apples, and* 2 *apples;* 4 *and* 3 *are* 7, *and* 2 *are* 9. *Therefore, they have in all* 9 *apples.*

2. Paid for paper eight cents, and for pencils 6; how much was paid for the whole?

3. James caught 7 fishes, Ernest 5, and Albert 6; how many were caught by them all?

4. Gave 9 dollars for coal, and 11 dollars for wood; how many dollars were given for both?

5. If a writing-book costs 10 cents, a pen 5 cents, and a slate 9 cents, how much do they all cost?

6. A clothier sold a hat for 5 dollars, a vest for 9, and a coat for as much as he sold both the hat and vest for; for how much was the coat sold?

7. Jane has 5 books at school, 7 at home, and has loaned 6; how many has she in all?

8. In one pasture are 10 sheep, in another 7, and in a third 8; how many are there in all?

9. A farmer had 27 cows and bought 8 more; how many had he then?

REVIEW. — *What is the* method of expressing numbers by figures *called? (32.)*

10. In a garden are 30 peach-trees, 7 pear-trees, and 9 apple-trees; how many trees are there in the garden?

11. How many are 33 and 29?

SOLUTION. — *As many as the sum of* 33 *and* 29; 33 *is* 3 *tens and* 3 *units, and* 29 *is* 2 *tens and* 9 *units;* 3 *tens and* 2 *tens are* 5 *tens, and* 3 *units and* 9 *units are* 12 *units, or* 1 *ten and* 2 *units;* 5 *tens and* 1 *ten are* 6 *tens, or* 60 *units; and* 60 *units and* 2 *units are* 62. *Therefore* 33 *and* 29 *are* 62.

12. How many are 25 and 35? 35 and 25?

13. How many are $20 + 23 + 16$? $16 + 23 + 20$?

14. A lady paid 55 dollars for a carpet, 20 dollars for a chair, and 15 dollars for a table; how much did she pay for the whole?

A PRINCIPLE OF ADDITION.

41. *The sum of numbers is the same in whatever order they are added.*

WRITTEN EXERCISES.

42. 1. Required the sum of 236, 541, and 102.

OPERATION.

$$\begin{array}{r} 236 \\ 541 \\ 102 \\ \hline \text{Sum, } 879 \end{array}$$

For convenience, we write the given numbers so that all the figures of the same order stand in the same column, and begin with units to add.

2, 1, and 6 units are 9 *units*, which we write for the units of the sum.

0, 4, and 3 tens are 7 *tens*, which we write for the tens of the sum.

1, 5, and 2 hundreds are 8 *hundreds*, which we write for the hundreds of the sum. The sum required is 8 hundreds 7 tens 9 units, or 879.

How are the given numbers written in the operation?

2. Required the sum of 595, 361, and 723.

OPERATION.

595
361
723
Sum, 1679

For convenience we write, as before, the figures of the same order in the same column, and begin with units to add.

3, 1, and 5 units are 9 *units;* we write the 9.

2, 6, and 9 are 17 *tens*, or 1 hundred and 7 tens; we write the 7, and add the 1 in with hundreds.

1, 7, 3, and 5 hundreds are 16 *hundreds*, or 1 thousand and 6 hundreds, which we write.

The sum is 1 thousand 6 hundreds 7 tens 9 units, or 1679.

In practice, it is sufficient to name only results. Thus,

In the operation we may say: three, four, *nine*, — write 9; two, eight, *seventeen*, — write 7 and add 1 with next column; eight, eleven, *sixteen*, — write 16; answer, 1679.

Rule. — *Write the numbers to be added so that figures of the same order shall stand in the same column.*

Beginning with units, add each column separately, and write the sum underneath, if less than ten of the order added.

If, however, the sum is ten or more, write the right-hand figure, and add the number expressed by the other figure or figures with the numbers of the next column.

Write the whole sum of the last column.

PROOF. — Add the numbers a second time, but so that they shall have been added the one time upwards and the other downwards.

Why is it convenient to have figures of the same order in the same column? *Ans.* Because only units of the same order can be directly added.

How do you write the numbers to be added? How do you add *each column?* °*Recite* the Rule. The Proof.

Examples.

(3.)	(4.)	(5.)	(6.)	(7.)
216	452	501	691	334
103	123	120	10	160
520	211	17	9	123
839	786	638	710	617

(8.)	(9.)	(10.)	(11.)	(12.)
610	317	491	216	929
291	812	870	725	442
208	402	143	140	312

(13.)	(14.)	(15.)	(16.)	(17.)
621	723	991	31	818
333	451	61	615	98
998	689	781	123	3

(18.)	(19.)	(20.)	(21.)	(22.)
110	72	402	331	928
99	491	40	617	713
6	149	449	100	456

How is addition proved? What is the reason for this proof? *Ans.* The same error, if any, is not likely to be repeated, when the same numbers are added in a different order.

Why do we find it convenient to begin with units to add? *Ans.* Because, if the sum of any column is greater than nine of the order added, we can readily add the number denoted by its left hand figure or figures with the next column.

(23.)	(24.)	(25.)	(26.)
1781	206	125	9020
121	312	39	3451
5642	871	620	7770
2401	462	510	1223

(27.) Dollars.	(28.) Years.	(29.) Miles	(30.) Bushels.
1234	7261	2168	2341
5610	345	1681	5123
1432	69	2002	4402

31. What is the sum of 9401, 105, and 47?

Ans. 9553.

32. What is the sum of $324 + 643 + 201 + 46$?

Ans. 1214.

33. What is the sum of $5684 + 340 + 7006 + 309 + 824$?

34. $170 + 640 + 126 + 123 + 11 =$ how many?

Ans. 1070.

35. $36 + 320 + 708 + 17 + 3 =$ how many?

Ans. 1084.

36. Find the sum of 31, 114, 3047, and 57183.

37. Find the sum of 19, 119, 11919, and 9191.

38. A farmer has four fields; the first yields 1109 bushels of corn, the second, 301, the third, 516, and the fourth 613; how many bushels do the four fields yield?

Ans. 2539 bushels.

39. George Washington was born in the year 1732, and lived 67 years. In what year did he die?

When the sum of a column is ten or more, why is the right-hand figure of the sum written underneath? *Ans.* Because it *expresses units of the order* added.

40. In January there are 31 days, February 28, March 31, April 30, May 31, and June 30; how many days are there in these six months? Ans. 181 days.

41. I have bought six lots of coal, consisting of 210, 317, 258, 610, 516, and 311 tons, respectively; how many tons are there in all? Ans. 2222 tons.

42. What is the sum of nine hundred three, four thousand fourteen, fifteen, sixty-three, and three thousand three hundred two? Ans. 8297.

43. What is the sum of fifteen thousand fifteen, one thousand six hundred one, ten thousand one hundred ten, and seven hundred seventeen? Ans. 27443.

44. Paid for a farm 6715 dollars, for having it fenced 635 dollars, and for having a barn built upon it 500 dollars. For how much must it be sold to gain 200 dollars?

45. What is the extent of the British Isles, if Scotland has 30000 square miles, England 51200, Wales 7200, and Ireland 32500? Ans. 120900 square miles.

46. The United States is 3385000 square miles in extent, Brazil 3000000, Russia in Europe 2100000, France 204800, and Austria 259300; what is the extent of all these countries? Ans. 8949100 square miles.

(47.)	(48.)	(49.)	(50.)
1141	310	451	8081
6131	709	723	7305
571	51	540	1941
450	600	172	1231
19	173	425	4207

Why is the number expressed by the left-hand figure or figures of the sum of a column added in with the next column at the left? *Ans.* Because ten units of the order added are one of the order next higher.

LONG COLUMNS.

Practise upon the following examples, until the numbers of each can be added correctly and rapidly, by naming results only.

(51.)	(52.)	(53.)	(54.)	(55.)
63	44	123	316	150
57	36	503	134	529
91	28	175	111	271
36	33	497	132	114
86	55	502	436	497
45	70	212	235	320
35	34	684	147	169
413		2696		2050

(56.)	(57.)	(58.)	(59.)	(60.)
307	21	746	20	173
133	86	672	93	813
810	24	336	18	324
341	14	187	78	407
458	43	267	34	611
817	54	164	27	710
802	73	928	57	206
863	32	748	26	502
4531		4048		3746

REVIEW. — What is a unit? (1.) What is Quantity? (10.) What are Numbers? (11.) What are Figures? (16.) What is Notation? (32.) What is Numeration? (36.) What is Addition? (39.) What is the result in Addition called? (40.) °Recite *the* Rule for Addition. (42.)

SUBTRACTION.

43. 1. If John should have 6 trouts and his brother 4, how many more will John have than his brother?

2. Six trouts less 4 trouts are how many?

3. Six less 4 are how many?

4. Four and how many are six?

44. **Subtraction** is the process of finding the difference between two numbers of the same kind.

The DIFFERENCE, or REMAINDER, is the number left after the subtraction is performed. Thus,

The difference between 8 apples and 6 apples is **2** apples.

The SUBTRAHEND is the number subtracted.

The MINUEND is the number subtracted from.

The DIFFERENCE between two numbers is such a number as added to the smaller will equal the larger.

What is Subtraction? The Difference? The Subtrahend? The Minuend? The Difference between two Numbers?

45. The **Sign of Subtraction** is a short horizontal line, —, and is called *minus*. Thus,

6 — 4, which is read six minus four, denotes that 4 is to be subtracted from 6.

8 — 5 = 3, denotes that 8 minus 5 is equal to 3, or that 8 less 5 is equal to 3.

Subtraction Tables.

1 from			2 from			3 from			4 from			5 from		
1	leaves	0	2	leaves	0	3	leaves	0	4	leaves	0	5	leaves	0
2	"	1	3	"	1	4	"	1	5	"	1	6	"	1
3	"	2	4	"	2	5	"	2	6	"	2	7	"	2
4	"	3	5	"	3	6	"	3	7	"	3	8	"	3
5	"	4	6	"	4	7	"	4	8	"	4	9	"	4
6	"	5	7	"	5	8	"	5	9	"	5	10	"	5
7	"	6	8	"	6	9	"	6	10	"	6	11	"	6
8	"	7	9	"	7	10	"	7	11	"	7	12	"	7
9	"	8	10	"	8	11	"	8	12	"	8	13	"	8
10	"	9	11	"	9	12	"	9	13	"	9	14	"	9
11	"	10	12	"	10	13	"	10	14	"	10	15	"	10

6 from			7 from			8 from			9 from			10 from		
6	leaves	0	7	leaves	0	8	leaves	0	9	leaves	0	10	leaves	0
7	"	1	8	"	1	9	"	1	10	"	1	11	"	1
8	"	2	9	"	2	10	"	2	11	"	2	12	"	2
9	"	3	10	"	3	11	"	3	12	"	3	13	"	3
10	"	4	11	"	4	12	"	4	13	"	4	14	"	4
11	"	5	12	"	5	13	"	5	14	"	5	15	"	5
12	"	6	13	"	6	14	"	6	15	"	6	16	"	6
13	"	7	14	"	7	15	"	7	16	"	7	17	"	7
14	"	8	15	"	8	16	"	8	17	"	8	18	"	8
15	"	9	16	"	9	17	"	9	18	"	9	19	"	9
16	"	10	17	"	10	18	"	10	19	"	10	20	"	10

What is the Sign of Subtraction? Recite the column 1 from 1. 2 from 2. 3 from 3. 4 from 4. 5 from 5. 6 from 6. 7 from 7. *8 from 8.* *9 from* 9. 10 from 10.

EXERCISES ON THE TABLES.

1. 2 from 6 how many?
2. 1 from 8 how many?
3. 3 from 7 how many?
4. 8 from 10 how many?
5. 6 from 11 how many?
6. 9 from 15 how many?
7. 3 from 11 how many?
8. 7 from 10 how many?
9. 3 from 8 how many?
10. 7 from 11 how many?
11. 6 from 12 how many?
12. 5 from 13 how many?
13. 9 from 13 how many?
14. 2 from 11 how many?
15. 7 from 12 how many?
16. 8 from 16 how many?

17. 6 from 8 how many?
18. 7 from 14 how many?
19. 3 from 6 how many?
20. 5 from 14 how many?
21. 6 from 13 how many?
22. 8 from 9 how many?
23. 4 from 4 how many?
24. 2 from 11 how many?
25. 5 from 17 how many?
26. 9 from 16 how many?
27. 2 from 10 how many?
28. 7 from 16 how many?
29. 9 from 18 how many?
30. 6 from 14 how many?

31. 13 — 4 = how many?
32. 9 — 5 = how many?
33. 13 — 5 = how many?
34. 16 — 6 = how many?
35. 17 — 10 = how many?
36. 18 — 9 = how many?
37. 8 — 2 = how many?
38. 14 — 8 = how many?
39. 11 — 1 = how many?
40. 16 — 7 = how many?
41. 19 — 10 = how many?
42. 20 — 10 = how many?

43. How many does 6 from 13 leave? 6 from 23? 6 from 43? 6 from 73? 6 from 93?

44. How many does 5 from 12 leave? 5 from 32? 5 from 22? 5 from 42? 5 from 72?

REVIEW. — What is the sign of Subtraction called? (45.) Sign of Addition? (40.)

45. How many does 7 from 11 leave? 7 from 21? 7 from 41? 7 from 64? 7 from 84?

46. How many does 8 from 16 leave? 8 from 36? 8 from 76? 8 from 96? 8 from 15? 8 from 55?

47. How many does 9 from 17 leave? 9 from 27? 9 from 37? 9 from 67? 9 from 67? 9 from 88?

48. How many does 3 from 10 leave? 3 from 40? 3 from 60? 3 from 70? 3 from 90?

MENTAL EXERCISES.

1. Mary had 8 books, and gave to her sister 3 of them; how many books had she left?

SOLUTION. — *She had left the difference between* 8 *books and* 3 *books, which is* 5 *books.*

2. If a boy has 13 marbles, and gives away 7 of them, how many has he left?

3. Bought a watch for 10 dollars and sold it for 17; how much was gained?

4. From a brood of 15 chickens the hawks have taken 6; how many remain?

5. If I go a-shopping with 18 cents, and spend 8 cents, how much have I left?

6. Sold goods for 21 dollars, which was five dollars more than they cost me; what did they cost me?

7. Bought a cow for 31 dollars, and paid down 9 dollars; how much was then owed?

8. Andrew has 4 cents, and wishes to buy a knife worth 42 cents; how many more cents does he need?

9. How many cents less than 42 cents are 4 cents?

REVIEW. — What are Numbers? (11.) Figures? (16.) How *many are used?* (16.)

9. 23 from 47 leaves how many?

SOLUTION. — *As many as the difference between* 47 *and* 23; 47 *is* 4 *tens and* 7 *units, and* 23 *is* 2 *tens and* 3 *units; 2 tens from* 4 *tens leaves* 2 *tens, and* 3 *units from* 7 *units leaves* 4 *units; and* 2 *tens, or* 20 *units, and* 4 *units are* 24. *Therefore,* 23 *from* 47 *leaves* 24.

10. 17 from 28 leaves how many? 19 from 39?

11. 25 from 46 leaves how many? 35 from 55?

12. There are 21 pupils in one school-room, and 33 in another; how many less are there in the one than in the other?

13. 58 chests of tea less 34 chests, are how many? 34 and how many are 58?

A PRINCIPLE OF SUBTRACTION.

46. *The difference and subtrahend, added together, equal the minuend.*

WRITTEN EXERCISES.

47. Subtract 325 from 958.

OPERATION.

Minuend,	958
Subtrahend,	325
Difference,	633

For convenience, we write the subtrahend under the minuend, so that figures of the same order stand in the same column, and begin at the right to subtract.

5 units from 8 units leave 3 units, which we write for the unit of the difference.

2 tens from 5 tens leave 3 tens, which we write for the tens of the difference.

3 hundreds from 9 hundreds leave 6 hundreds, which we write for the hundreds of the difference.

The difference required is 6 hundreds 3 tens 3 units, or 633.

How do you write the given numbers? Why is it convenient to have the figures of the subtrahend written under figures of the same order in the minuend? *Ans.* Because only units of the same order can be subtracted.

2. Find the difference between 652 and 423.

OPERATION.

Minuend,	652
Subtrahend,	423
Difference,	229

For convenience, we write, as before, figures of the same order in the same column, and begin with units to subtract.

We cannot take 3 units from 2 units; but we can take 1 ten from the 5 tens, leaving 4 tens; and the 1 ten taken is 10 units, which added to the 2 units make 12 units; 3 units from 12 units leave 9 units, which we write for the units of the difference.

2 tens from 4 tens leave 2 tens, which we write for the tens of the difference.

4 hundreds from 6 hundreds leave 2 hundreds, which we write for the hundreds of the difference.

Therefore, the difference required is 2 hundreds 2 tens 9 units, or 229.

The operation may be performed in another way: as the 3 units cannot be taken from 2 units, we add 10 units to the 2 units; and 3 units from 12 units leave 9 units. To balance the 10 units added to the 2 units, we add 1 ten to the 2 tens, in accordance with the principle, that *if any two numbers are equally increased their difference remains the same;* and say, 3 tens from 5 tens leave 2 tens.

4 hundreds from 6 hundreds leave 2 hundreds.

Therefore, the difference is 229.

In practice, a briefer explanation is sufficient. Thus,

We may say: 3 from 2, impossible; but 3 from 12 leaves 9; we write 9, and add 1 to the next figure; 1 plus 2 are 3, and 3 from 5 leaves 2, which we write; 4 from 6 leaves 2, which we write. Answer, 229.

Rule. — *Write the less number under the greater, so that units of the same order shall stand in the same column.*

When the number of an order of the subtrahend is greater than that of the corresponding order of the minuend, what do you do? Mention another way. According to what principle is the 10 added to an order of the minuend, balanced by the addition of 1 to the next left-hand order of the subtrahend?

Beginning with units, subtract each order of the subtrahend from the order above it, and write the difference underneath.

If the number of any order of the subtrahend be greater than that of the order above it, conceive the number of the upper order to be increased by ten, and the number of the next order of the minuend to be diminished by one, or the next order of the subtrahend increased by one, and proceed as before.

PROOF. — Add the difference to the subtrahend, and, if the work is correct, the sum will equal the minuend. Or,

Subtract the difference from the minuend, and if the work is correct, what is left will equal the subtrahend.

Examples.

	(3.)	(4.)	(5.)
Minuend,	466	673	2006
Subtrahend,	270	564	107
Difference,	196	109	1899
Proof,	466	673	2006

	(6.)	(7.)	(8.)	(9.)
From	890	526	812	723
Subtract	763	19	102	16

	(10.)	(11.)	(12.)	(13.)
From	423	902	876	200
Take	310	713	575	189

How are the given numbers written for Subtraction? Where do you begin to subtract? If the number denoted by the subtrahend is greater than that denoted by the figure above it, what do you do? Recite the Rule. What is the method of proof?

	(14.)	(15.)	(16.)	(17.)
From	333	900	776	307
Take	244	109	445	206
	(18.)	(19.)	(20.)	(21.)
From	2006	2110	1008	4708
Take	1341	1102	19	1080
	(22.) Dollars.	(23.) Miles.	(24.) Pounds.	(25.) Yards.
From	7987	4162	7910	8008
Take	890	1051	6820	909
	7097	3111	1090	7099

26. From 1937 tons subtract 1128 tons.

Ans. 809 tons.

27. From 1000 barrels take 98 barrels.

Ans. 902 barrels.

28. 463 cords less 209 cords = how many cords?

29. 3612 rods — 2004 rods = how many rods?

30. 21020 — 16101 = how many? Ans. 4919.

31. 30000 — 1010 = how many? Ans. 28990.

32. 111111 — 11112 = how many? Ans. 99999.

33. Subtract 635 from 34621. Ans. 33986.

34. Subtract 31012 from 31201. Ans. 189.

35. Take 416 from 928. Ans. 512.

36. Take 313142 from 404030. Ans. 90888.

37. I had 1630 dollars in the bank, but have drawn out 635 dollars. How many dollars have I remaining in the bank? Ans. 995 dollars.

REVIEW. — What is the sign of Subtraction? (45.) What is the sign of Equality? (40.)

38. Paid 225 dollars for a horse, and 165 dollars for a buggy; how much more was paid for the horse than for the buggy? Ans. 60 dollars.

39. How old was a man in 1865, who was born in 1819?

40. The Pilgrims landed at Plymouth in 1620; how long after was the Declaration of Independence, which was in 1776? Ans. 156 years.

41. John Jones bought a farm for 4000 dollars, and has paid for it within 719 dollars; how much has he paid already? Ans. 3281 dollars.

42. Maryland has 9356 square miles, and Delaware 2120 square miles; how many more square miles has the one than the other? Ans. 7236.

43. A farmer raised 6723 bushels of corn, and has sold 3846 bushels; how much remains unsold?

44. From ten thousand ten, take one thousand one hundred and one. Ans. 8909.

45. From sixteen thousand five hundred and ninety, take twelve thousand nine hundred and nine. Ans. 3681.

46. A trader bought 30 casks of cider, containing 1890 gallons, and sold 23 casks, containing 1449 gallons. How many casks, and how many gallons, were left?

Ans. 7 casks; 441 gallons.

47. The salary of the President in four years amounts to two hundred thousand dollars; if his expenses in that time are 63575 dollars, how much of his salary will be left? Ans. 136425 dollars.

48. The smaller of two numbers is 61342, and the larger 1000300; what is their difference?

REVIEW. — What is Addition? (39.) What is the Sum? (39.) Of what name or kind is the sum? *Ans.* The same as that of the *numbers added.*

49. Albert Smith sold in one year goods to the amount of 58692 dollars, which was 5863 dollars more than he gave for them. How much did he give for the goods?

Ans. 52829 dollars.

50. If the mean distance of the sun from the earth is ninety-five million one hundred seventy-three thousand one hundred twenty-seven miles, and that of the moon two hundred thirty-eight thousand six hundred fifty miles, how much farther is the sun than the moon from the earth?

REVIEW.

1. A man having 7839 dollars, paid away at one time 317 dollars, and at another 2163; what did he then have left?

OPERATION.

2163	7839
317	2480
2480	5359

He had left the difference between 7839 dollars and the sum of 2163 dollars and 317 dollars; the sum of 2163 dollars and 317 dollars is 2480 dollars; and the difference between 7839 dollars and 2480 dollars is 5359 dollars. He had left 5359 dollars.

2. Bought flour to the amount of 256 dollars, cloth to the amount of 317 dollars, and paid down 373 dollars. How much remained to be paid?

3. Henry Smith went out with 435 dollars to settle accounts. He paid 115 dollars to one man, received 119 dollars from another, and paid 316 to another. How much had he left? Ans. 123 dollars.

4. A boy had 216 apples; he sold 34 of them one day, 72 another day, and afterwards 29; how many had he left?

REVIEW. — What is Subtraction? How does it differ from Addition? *Ans.* It is the reverse of Addition.

5. Bought a house for 3165 dollars, laid out for repairs on it 593 dollars, and sold it for 3500 dollars. How much was lost by the bargain? Ans. 258 dollars.

6. Sold cloth for 10 dollars, sugar 17 dollars, pork 43 dollars, beef 19 dollars, and fish 13 dollars; and in payment received a check for 75 dollars, and a 50-dollar note. How much change should be paid back? Ans. 23 dollars.

7. A farmer buys a yoke of oxen for 163 dollars. He exchanges them for others, paying 27 dollars, and then sells for 200 dollars. Did he gain or lose? — and how much?

8. Dana estimates that there are workable coal measures in the United States to the extent of 130000 square miles; and in Great Britain and Ireland of 12000, in Spain of 4000, in France of 2000, and in Belgium of 518. How much do those of the United States exceed those of the other countries named? Ans. 111482 square miles.

MULTIPLICATION.

48. 1. If a boy can earn 5 dollars in one week, how many dollars can he earn in 4 weeks?

2. How many are 4 times 5?

3. Susan has 6 books and Mary has 3 times as many. How many books has Mary?

4. How many are 3 times 6?

5. If you obtain 7 merit marks in 1 day, how many will you obtain in 6 days?

Of what name or kind is the answer in Subtraction? *Ans.* The same as that of the Minuend and Subtrahend. What is the Rule for Subtraction? Proof?

49. Multiplication is the process of taking one number as many times as there are units in another number.

The PRODUCT is the result of the multiplication.

The MULTIPLICAND is the number to be multiplied.

The MULTIPLIER is the number by which the multiplicand is to be multiplied.

50. The **Factors** are the multiplier and multiplicand.

Multiplication Tables.

Once	2 times	3 times	4 times	5 times	6 times
1 is 1	1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 2	2 " 4	2 " 6	2 " 8	2 " 10	2 " 12
3 " 3	3 " 6	3 " 9	3 " 12	3 " 15	3 " 18
4 " 4	4 " 8	4 " 12	4 " 16	4 " 20	4 " 24
5 " 5	5 " 10	5 " 15	5 " 20	5 " 25	5 " 30
6 " 6	6 " 12	6 " 18	6 " 24	6 " 30	6 " 36
7 " 7	7 " 14	7 " 21	7 " 28	7 " 35	7 " 42
8 " 8	8 " 16	8 " 24	8 " 32	8 " 40	8 " 48
9 " 9	9 " 18	9 " 27	9 " 36	9 " 45	9 " 54
10 " 10	10 " 20	10 " 30	10 " 40	10 " 50	10 " 60
11 " 11	11 " 22	11 " 33	11 " 44	11 " 55	11 " 66
12 " 12	12 " 24	12 " 36	12 " 48	12 " 60	12 " 72

7 times	8 times	9 times	10 times	11 times	12 times
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 14	2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 21	3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 28	4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 35	5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 42	6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 49	7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 56	8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 63	9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 70	10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 77	11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 84	12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

What is Multiplication? The Product? The Multiplicand? *The Multiplier?* The Factors? Recite the Tables.

51. The **Sign of Multiplication** is an inclined cross, $\times$, and is read *multiplied by*.

Thus, 5×4 is read, 5 multiplied by 4.

EXERCISES ON THE TABLES.

1. 3 times 8 are how many?
2. 2 times 7 are how many?
3. 4 times 2 are how many?
4. 5 times 3 are how many?
5. 4 times 5 are how many?
6. 7 times 3 are how many?
7. 6 times 7 are how many?
8. 6 times 2 are how many?
9. 0 times 9 are how many?
10. 8 times 3 are how many?
11. 9 times 7 are how many?
12. 7 times 0 are how many?
13. 5 times 8 are how many?
14. 6 times 5 are how many?

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15. $8 \times 9 =$ how many?
16. $10 \times 2 =$ how many?
17. $9 \times 0 =$ how many?
18. $7 \times 2 =$ how many?
19. $8 \times 4 =$ how many?
20. $11 \times 3 =$ how many?
21. $12 \times 5 =$ how many?
22. $3 \times 9 =$ how many?
23. $12 \times 2 =$ how many?
24. $4 \times 7 =$ how many?
25. $11 \times 5 =$ how many?
26. $9 \times 6 =$ how many?
27. $10 \times 3 =$ how many?
28. $7 \times 12 =$ how many?

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29. 6 times 11 are how many?
30. 7 times 7 are how many?
31. 5 times 10 are how many?
32. 4 times 9 are how many?
33. 11 times 10 are how many?
34. 8 times 8 are how many?
35. 9 times 9 are how many?
36. 11 times 11 are how many?
37. 10 times 10 are how many?
38. 12 times 12 are how many?

MENTAL EXERCISES.

1. What will 3 oranges cost at 6 cents apiece?

SOLUTION. — *At* 6 *cents apiece*, 3 *oranges will cost* 3 *times* 6 *cents*, *or* 18 *cents*.

2. If a boat will carry 7 men, how many men will 4 boats of the same size carry?

What is the sign of Multiplication?

3. If a quart of berries can be bought for 8 cents, how many cents will be required to buy 6 quarts?

4. How much will 8 melons bring, at 12 cents each?

5. By reading 9 pages a day, how many pages can you read in 9 days?

6. What cost 12 tons of coal, at 5 dollars a ton?

7. What cost 11 pencils, at 10 cents apiece?

8. If a boat sails at the rate of 9 miles an hour, how far will it sail in 12 hours?

9. When flour is 8 dollars a barrel, how much must be paid for 7 barrels?

10. How many are 4 times 24?

SOLUTION. — *As many as the product of* 24 *by* 4. 24 *is* 2 *tens* 4 *units; 4 times* 2 *tens are* 8 *tens, or* 80 *units, and* 4 *times* 4 *units are* 16 *units; which added to* 80 *units equal* 96 *units. Therefore,* 4 *times* 24 *are* 96.

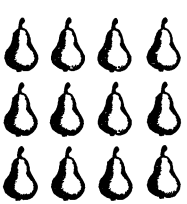
11. How many are 3 times 16? 3 times 35?

12. How many are 5 times 18? 4 times 27?

13. What cost 7 pounds of butter, at 41 cents a pound?

A PRINCIPLE OF MULTIPLICATION.

52. *The product of two or more factors is the same in whatever order they are taken.*



For the product of 4 by 3, or 3 by 4, is 12. This is shown by the illustration. The number of pears in the picture are the same, whether we consider them 4 in a row and 3 rows, or 3 in a row and 4 rows.

Show that the product of two factors is the same in whatever *order* taken.

WRITTEN EXERCISES.

53. Multiply 564 by 7.

OPERATION.

Multiplicand, 564
Multiplier, 7
Product, 3948

For convenience, we write the multiplier under the units in the multiplicand, and begin with units to multiply.

7 times 4 units are 28 units, which equal 2 tens 8 units; we write the 8 units, for the units of the product, and reserve the 2 tens.

7 times 6 tens are 42 tens, which with the 2 tens added are 44 tens, or 4 hundreds 4 tens; we write the 4 tens, and reserve the 4 hundreds.

7 times 5 hundreds are 35 hundreds, which with the 4 hundreds added are 39 hundreds, or 3 thousands 9 hundreds; which we write.

Therefore, the product is 3 thousands 9 hundreds 4 tens 8 units, or 3948.

2. Multiply 736 by 206.

OPERATION.

Multiplicand, 736
Multiplier, 206
Partial Products, 4416, 14720
Product, 151616

For convenience, we write the multiplier under the multiplicand, so that figures of the same order stand in the same column; and multiplying by the units, as in the preceding operation, we obtain 4416 units.

There being 0 tens, we write 0 tens, and pass to the hundreds' order of the multiplier.

2 hundreds are two hundred units, and 2 hundred times 6 units are 12 hundreds, or 1 thousand 2 hundreds. We write the 2 hundreds, and reserve the 1 thousand.

2 hundred times 3 tens are 6 hundred tens, equal 6 thousands, and the 1 thousand added are 7 thousands, which we write.

Why is it convenient to begin with units to multiply? *Ans.* Because, if the product of any of the orders contains units of the next higher order, they may be readily added to the product of that order.

2 hundred times 7 hundreds are 14 hundred-hundreds, equal 1 hundred-thousand 4 ten-thousands, which we write; and obtain 1472 hundreds, or, with the 0 tens on the right, 14720 tens Adding the two partial products we have for the entire product 151616. Therefore, the product of 736 by 206 is 151616.

In practice, the names of the orders of units may be omitted. Thus, in the operation of the first question: 7 times 4 are 28; we write the 8, and add the 2 to the next product; 7 times 6 are 42, and 2 are 44; we write 4, and add 4 to the next product; 7 times 5 are 35, and 4 are 39; answer, 3948.

Rule. — *Write the multiplier under the multiplicand, so that units may stand under units, tens under tens, etc.*

If the multiplier contains but one order, begin with units and multiply each order of the multiplicand by it. Write the right-hand figure of each product underneath, add its other orders, if any, to the next product, and observe to write all the figures of the last product.

If the multiplier contains more than one order, multiply by each order separately, writing the right-hand figure of each partial product under the order used. The sum of the partial products will be the entire product.

PROOFS. — Multiply the multiplier by the multiplicand; and, if the work is right, the product will be the same as that first obtained. Or,

Multiply the multiplicand by a multiplier one less than the given multiple, and add the multiplicand to the product.

Why is the right-hand figure of each partial product written under its multiplier? *Ans.* That units of the same order may stand in the same column, for convenience in adding. °Recite the Rule. *What are the Proofs?*

Examples.

3. Find the product of 534 by 242.

534				242
242				534
1068	= 534 × 2			968
2136	= 534 × 40		*Proof.*	726
1068	= 534 × 200			1210
Ans. 129288	= 534 × 242			129228

	(4.)	(5.)	(6.)	(7.)
Multiply	361	723	892	904
By	5	8	9	4
Ans.	1805	5784	8028	3616

(8.)	(9.)	(10.)	(11.)	(12.)
707	335	806	139	678
7	3	6	2	8

(13.)	(14.)	(15.)	(16.)
1031	2461	3008	1213
11	5	4	12

(17.)	(18.)	(19.)	(20.)
317	1426	1260	2371
13	15	14	23

Why is a partial product of the same order of units as the figure of the multiplier which produced it? *Ans.* Because units multiplied by *units* produces *units*; units multiplied by *tens* produces *tens*; units multiplied by *hundreds* produces *hundreds*, etc.

(21.)	(22.)	(23.)	(24.)
1617	1212	8023	1441
121	192	66	315

25. Multiply 956 by 12. Ans. 11472.
26. Multiply 527 by 7. Ans. 3689.
27. Multiply 365 by 25. Ans. 9125.
28. Multiply 1836 by 32. Ans. 58752.
29. Multiply 895 by 11. Ans. 9845.
30. 3706 × 207 = how many? Ans. 767142.
31. 2135 × 19 = how many? Ans. 40565.
32. 958 × 34 = how many? Ans. 32572.
33. 47696 × 144 = how many? Ans. 6868224.

54. Since the product of any number by 0 is 0, observe to write in an operation 0 under each 0 of the multiplier.

If, however, the multiplier is 10, 100, 1000, etc., for the product, write after the multiplicand as many ciphers as are in the multiplier.

For, the value expressed by figures is made tenfold by each removal of them an order to the left. (Art. 34.) Thus 2 × 10 = 20, 2 × 100 = 200, etc.

	(34.)	(35.)	(36.)	(37.)
Multiply	519	785	162	2400
By	20	60	103	200
Ans.	10380			480000

Why do you write a cipher in the operation under each cipher of the multiplier? How do you multiply by 10, 100, 1000, etc ? Give the reason.

38. Multiply 23467 by 10. Ans. 234670.

39. Multiply 68920 by 100.

40. Multiply 48005 by 3040. Ans. 145935200.

41. How much will a farm of 149 acres cost, at 96 dollars an acre? Ans. 14304 dollars.

42. If a ship sail 192 miles a day, how far will it sail in 56 days? Ans. 10752 miles.

43. If a coach-wheel turn round 346 times in going a mile, how many times will it turn round in going 95 miles? Ans. 32870 times.

44. What will 49 yards of drilling cost, at 37 cents per yard? Ans. 1813 cents.

45. What will 145 tons of hay come to, at 15 dollars a ton? Ans. 2175 dollars.

46. How much will 320 barrels of flour cost, at 9 dollars a barrel? Ans. 2880 dollars.

47. How many men are there in an army of 20 regiments, of 980 men each? Ans. 19600.

48. What is the value of 130 United States bonds, at 113 dollars each? Ans. 14690 dollars.

49. If the average number of beats of the heart in childhood is 105 in a minute, what will be the number per hour, or 60 minutes? Ans. 6300.

50. If the earth in its orbit around the sun moves at the rate of 68000 miles an hour, how far does it move in one day, or 24 hours? Ans. 1632000 miles.

51. Light travels 192000 miles in 1 second; how far will it travel in 8 minutes of 60 seconds each?

Ans. 92160000 miles.

REVIEW. — What is Addition? (39.) Subtraction? (44.) What is the result of Addition called? (39.) What is the result of Subtraction called? (44.)

REVIEW.

1. If 37 trees yield 13 bushels of apples each, and 41 others yield 11 bushels each, how many bushels do they all yield?

SOLUTION. — *They all yield as many bushels as* 37 *times* 13 *bushels, and* 41 *times* 11 *bushels.*

37 *times* 13 *bushels are* 481 *bushels, and* 41 *times* 11 *bushels are* 451 *bushels; and* 481 *bushels and* 451 *bushels are* 932 *bushels. Therefore, they all yield* 932 *bushels.*

2. Bought 713 tons of coal for 2852 dollars, and sold it at 6 dollars a ton; how much was gained by the operation? Ans. 1426 dollars.

3. Sold 325 barrels of apples at 3 dollars a barrel, and received 650 dollars in cash, and a note for the balance. For what sum was the note?

4. A farmer bought 32 sheep at 6 dollars each, 31 cows at 45 dollars each, and 3 horses at 162 dollars each. He sold the whole for 2500 dollars; how much did he gain? Ans. 427 dollars.

5. A merchant bought 7 rolls of carpeting, each containing 27 yards, and 4 rolls each containing 18 yards. How many yards did he buy in all? Ans. 261 yards.

6. If a man earn 950 dollars a year, and his expenses are 2 dollars a day, how much can he save in a year, or 365 days?

7. Jones and Smith start from the same place and travel in opposite directions, Jones at the rate of 32 miles a day, and Smith at the rate of 34 miles a day. How far apart will they be at the end of 6 days? Ans. 396 miles.

REVIEW. — What is the larger of the two given numbers called in Subtraction? *Ans.* The Minuend. In Multiplication, what is the result called? (49.)

8. Arthur has 135 dollars and his brother has 4 times as much, lacking 15 dollars; how much has his brother?

Ans. 525 dollars.

9. Bought a dictionary for 6 dollars, a set of histories for 22 dollars, and a select library for 137 dollars; and gave in payment a check for 63 dollars, and three 50-dollar bills. How much should be received back?

Ans. 48 dollars.

10. Bought one farm of 63 acres at 105 dollars per acre, and another of 240 acres at 95 dollars per acre. What did both farms cost me?

11. Bought 31 parlor organs at 145 dollars each; and gave in payment 21 United States bonds at 111 dollars each, four 500-dollar bills, and my note for the balance. For what sum did I give my note? Ans. 164 dollars.

12. Sold 560 tons of coal at 9 dollars a ton, and received in payment 875 bushels of wheat at 2 dollars per bushel, and the balance in money. How much money did I receive?

13. A merchant had in a bank 8000 dollars, drew out 7563 dollars, and afterwards put in 1550 dollars; how much had he then in the bank? Ans. 1987 dollars.

14. Two steamers start at the same time to meet each other, the one from Boston and the other from Liverpool, which places are 2883 miles apart. Provided that one sails at the rate of 192 miles a day, and the other at the rate of 168 miles a day, how many miles will they be from each other at the end of 5 days? Ans. 1083 miles.

REVIEW. — What is Multiplication? (49.) The Product? (49.) The Multiplicand? (49.) The Multiplier? (49.)

DIVISION.

55. 1. At 2 cents each, how many apples can be bought for 8 cents?

2. How many times 2 cents are there in 8 cents?

3. How many times 2 cents are 8 cents?

4. If 9 marbles be divided equally among 3 boys, how many marbles will each boy have?

5. Into how many equal parts of 3 marbles each, can 9 marbles be separated?

6. How many times 3 marbles are 9 marbles?

7. What is one of the 3 equal parts of 9 marbles?

8. What is one of the 4 equal parts of 8 cents?

9. If 4 barrels be a dray load, how many dray loads will be 12 barrels?

10. How many times 4 barrels are 12 barrels?

11. How many times 3 barrels are 12 barrels?

12. How many times 4 in 12? 3 in 12?

Review. — How do you prove Addition? (42.) How do you prove Subtraction? (47.) How do you prove Multiplication? (53.)

56. One of *two* equal parts of a number is called **one half.** Thus, one half of two is one.

One of *three* equal parts of a number is called **one third.** Thus, one third of six is two.

One of *four* equal parts of a number is called **one fourth.** Thus, one fourth of four is one.

In like manner, one of the five equal parts of a number is called **one fifth;** one of the six equal parts is called **one sixth;** one of the ten equal parts is called **one tenth,** etc. Hence,

57. A Whole Number has two halves, three thirds, four fourths, five fifths, ten tenths, etc.

58. One or more equal parts of a unit are called **Fractions,** to distinguish them from unbroken or whole numbers, which are called **Integers,** or **Integral Numbers.**

59. Division is the process of finding how many times one number is contained in another; or of finding one of the equal parts of a number.

The DIVIDEND is the number to be divided.

The DIVISOR is the number by which we divide.

The QUOTIENT is the result or number obtained by the division.

60. The REMAINDER is that part of the dividend which is left after finding the exact whole number of the quotient. Thus,

3 is contained in 7, 2 times and 1 remainder. The division of this 1, indicated, gives $\frac{1}{3}$, which, written after

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What is one of the two equal parts of a number called? One of three equal parts? One of four equal parts? One of five? One of six? What is Division? What is the Dividend? The Divisor? The Quotient? The Remainder?

the whole number of the quotient, makes the complete quotient $2\frac{1}{3}$, which is read *two and one-third.*

Division Tables.

1 in	2 in	3 in	4 in	5 in
1, 1 time	2, 1 time	3, 1 time	4, 1 time	5, 1 time
2, 2 times	4, 2 times	6, 2 times	8, 2 times	10, 2 times
3, 3 "	6, 3 "	9, 3 "	12, 3 "	15, 3 "
4, 4 "	8, 4 "	12, 4 "	16, 4 "	20, 4 "
5, 5 "	10, 5 "	15, 5 "	20, 5 "	25, 5 "
6, 6 "	12, 6 "	18, 6 "	24, 6 "	30, 6 "
7, 7 "	14, 7 "	21, 7 "	28, 7 "	35, 7 "
8, 8 "	16, 8 "	24, 8 "	32, 8 "	40, 8 "
9, 9 "	18, 9 "	27, 9 "	36, 9 "	45, 9 "
10, 10 "	20, 10 "	30, 10 "	40, 10 "	50, 10 "

6 in	7 in	8 in	9 in	10 in
6, 1 time	7, 1 time	8, 1 time	9, 1 time	10, 1 time
12, 2 times	14, 2 times	16, 2 times	18, 2 times	20, 2 times
18, 3 "	21, 3 "	24, 3 "	27, 3 "	30, 3 "
24, 4 "	28, 4 "	32, 4 "	36, 4 "	40, 4 "
30, 5 "	35, 5 "	40, 5 "	45, 5 "	50, 5 "
36, 6 "	42, 6 "	48, 6 "	54, 6 "	60, 6 "
42, 7 "	49, 7 "	56, 7 "	63, 7 "	70, 7 "
48, 8 "	56, 8 "	64, 8 "	72, 8 "	80, 8 "
54, 9 "	63, 9 "	72, 9 "	81, 9 "	90, 9 "
60, 10 "	70, 10 "	80, 10 "	90, 10 "	100, 10 "

61. The **Sign of Division** is a short horizontal line with a dot above and below it, $\div$, or a curved line,), and is read, *divided by.* Thus,

$6 \div 2$ is read, 6 divided by 2; and 2)6 is read, 6 divided by 2, or 2 in 6.

What is the Sign of Division? Recite the column 1 in 1. 2 in 2. 3 in 3. 4 in 4. 5 in 5. 6 in 6. 7 in 7. 8 in 8. 9 in 9. 10 in 10.

Sometimes, in place of the dots, the number divided is written above the line, and the number which divides it is written below. Thus,

$\frac{6}{2}$ is read, six divided by two, or six halves.

EXERCISES ON THE TABLES.

1. 4 in 12 how many times?
2. 5 in 10 how many times?
3. 3 in 27 how many times?
4. 8 in 64 how many times?
5. 2 in 16 how many times?
6. 6 in 42 how many times?
7. 9 in 18 how many times?
8. 3 in 18 how many times?
9. 7 in 49 how many times?
10. 4 in 16 how many times?
11. 5 in 50 how many times?
12. 8 in 64 how many times?
13. 9 in 54 how many times?
14. 10 in 70 how many times?

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15. $8 \div 2 =$ how many?
16. $18 \div 6 =$ how many?
17. $27 \div 3 =$ how many?
18. $56 \div 7 =$ how many?
19. $20 \div 2 =$ how many?
20. $36 \div 6 =$ how many?
21. $81 \div 9 =$ how many?
22. $30 \div 3 =$ how many?
23. $15 \div 5 =$ how many?
24. $24 \div 8 =$ how many?
25. $45 \div 9 =$ how many?
26. $40 \div 5 =$ how many?
27. $63 \div 7 =$ how many?
28. $72 \div 8 =$ how many?

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29. 3 in 30 how many times?
30. 5 in 35 how many times?
31. 7 in 28 how many times?
32. 8 in 16 how many times?
33. 7 in 14 how many times?
34. 9 in 63 how many times?
35. 10 in 80 how many times?
36. 2 in 18 how many times?
37. 6 in 54 how many times?
38. 4 in 36 how many times?
39. 10 in 90 how many times?
40. 8 in 80 how many times?
41. 9 in 72 how many times?
42. 10 in 100 how many times?

What sometimes takes the place of the dots in the sign of Division?

MENTAL EXERCISES.

1. How many oranges, at 5 cents each, can be bought for 45 cents?

SOLUTION. — *As many oranges, as 5 cents are contained times in 45 cents, which are 9. Therefore, 9 oranges can be bought for 45 cents.*

2. If hats are 4 dollars each, how many can be bought for 24 dollars?

3. How many quarts of milk, at 6 cents a quart, can be bought for 42 cents?

4. When berries are 10 cents a quart, how many quarts can be bought for 60 cents?

5. At 7 dollars a ton, how many tons of coal can be bought for 49 dollars?

6. How many pounds of sugar, at 9 cents a pound, can be bought for 72 cents?

7. If you can travel 4 miles in an hour, how many hours will it take you to travel 36 miles?

8. When 3 barrels of flour cost 27 dollars, what will be the cost of 1 barrel?

SOLUTION. — *The cost of 1 barrel will be one-third of the cost of 3 barrels; and one-third of 27 dollars is 9 dollars. Therefore, the cost of 1 barrel will be 9 dollars.*

9. If 7 men cut 42 cords of wood in a week, how many cords does 1 man cut?

10. If 10 pounds of rice cost 90 cents, how much is it a pound?

11. A farmer sold eight sheep for 40 dollars; how much apiece did he get for them?

REVIEW. — When a number is divided into two equal parts, what *is one of* the parts called? When divided into three equal parts?

12. At 30 dollars for 3 plows, what is the cost of each plow?

13 Mr. Rollins bought 9 barrels of flour for 72 dollars; what did he pay per barrel?

14. How many times is 8 contained in 96?

SOLUTION. — 96 *is* 9 *tens* 6 *units*; 8 *in* 9 *tens*, 1 *ten time, and* 1 *ten remaining*; 1 *ten remaining equals* 10 *units*; 10 *units and* 6 *units are* 16 *units*; *and* 8 *in* 16 *units* 2 *units times. Therefore,* 8 *is contained in* 96, 1 *ten and* 2 *units times, or* 12 *times.*

15. How many times is 5 contained in 65?

16. Divide 99 by 3. 88 by 4. 77 by 7.

17. How many times 5 in 125? In 175?

18. How many times 6 in 132? In 108?

19. If 9 writing-books cost 108 cents, what is the cost of each?

20. At 104 dollars for 8 tons of coal, what is the cost per ton?

21. At 7 cents a pound, what will 11 pounds of rice cost?

PRINCIPLES OF DIVISION.

62. 1. *Division is the reverse of multiplication.*

For the *dividend* answers to the *product*, and the *divisor* and *quotient* to the *factors*. Hence,

2. *The product of the quotient and divisor is equal to the dividend.*

Of what is Division the reverse? What does prefixed mean? *Ans.* Placed before.

SHORT DIVISION.

63. Division is called **Short Division** when the work of the solution is not written, except the answer.

64. 1. Divide 1701 by 4.

OPERATION.

Divisor, 4)1701 *Dividend.*

$425\frac{1}{4}$ *Quotient.*

For convenience we write the divisor at the left of the dividend, and begin at the left to divide.

4 is contained in 17 hundreds 4 hundreds times and 1 hundred remaining. We write the 4 hundreds in the quotient.

The 1 hundred we consider prefixed to the 0 tens, making 10 tens. 4 in 10 tens, 2 tens times, and 2 tens remaining. We write the 2 tens in the quotient.

The remaining 2 tens we consider prefixed to the 1 unit, making 21 units. 4 in 21 units, 5 units times, and 1 unit remaining. We write the 5 units in the quotient.

We then indicate the division of the remainder, 1 unit by the divisor 4, and annex to the integral part of the quotient; and have for the complete quotient $425\frac{1}{4}$.

Therefore, the quotient of 1701 divided by 4 is $425\frac{1}{4}$.

2. Divide 763 by 7.

OPERATION.

Divisor, 7)763 *Dividend.*

109 *Quotient.*

For convenience we write the divisor and begin to divide as in the preceding operation.

7 in 7 hundreds, 1 hundreds time, which we write in the quotient.

7 in 6 tens, 0 tens time, which we write in the quotient.

We prefix the undivided 6 tens to the 3 units, making 63 units. 7 in 63 units, 9 times, which we write in the quotient.

Therefore, the quotient of 763 divided by 7 is 109.

In practice we may say, 7 in 7, 1, which we write; 7 in 6, 0, which we write; prefix the 6 to the figure 3; 7 in 63, 9, which we write; answer, 109.

When is the division called Short Division?

Rule. — *Write the divisor at the left of the dividend.*

Begin at the left; divide the number expressed by the fewest figures of the dividend that will contain the divisor, and write the result beneath.

If there be a remainder, regard it as prefixed to the figure of the next lower order; divide as before, and so continue till all the figures of the dividend have been used.

If any partial dividend be less than the divisor, write a cipher in the quotient, and prefix the partial dividend to the figure of the next lower order, if any, for a new dividend.

If there be a final remainder, write it, with the divisor beneath, after the integral part of the quotient.

PROOF. — Multiply the integer of the quotient by the divisor, and to the product add the remainder, if any; and the result will equal the dividend, if the work is right.

Examples.

(3.)		(4.)	(5.)
Divisor, 5)1805	*Dividend.*	3)694	9)1890
361	*Quotient.*	*Ans.* $231\frac{1}{3}$	*Ans.* 210
5		3	9
Proof, 1805		*Proof*, 694	*Proof*, 1890

(6.)	(7.)	(8.)	(9.)
4)8126	7)3514	9)3464	3)1005
$2031\frac{2}{4}$	502	$384\frac{8}{9}$	335

How do you write the divisor? Where do you begin to divide? How do you proceed if there is a remainder? When any partial dividend is less than the divisor? When there is a remainder after dividing the last figure? What is the Rule? The Proof?

(10.)	(11.)	(12.)	(13.)
7)813	6)1705	9)9100	4)3787
(14.)	(15.)	(16.)	(17.)
7)80971	3)5280	8)90374	9)91080

18. Divide 57848 by 8. Ans. 7231.
19. Divide 8028 by 9. Ans. 892.
20. Divide 3616 by 4. Ans. 904.
21. Divide 2275 by 7. Ans. 325.
22. 50405 ÷ 5 = how many? Ans. 10081.
23. 65412 ÷ 3 = how many? Ans. 21804.
24. 947 ÷ 4 = how many? Ans. $236\frac{3}{4}$.
25. $54\frac{6}{3}36$ = how many? Ans. 18212.

65. When the divisor is 10, 100, 1000, etc., the quotient is obtained, at once, by cutting off at the right of the dividend as many figures as there are ciphers in the divisor.

For, since figures are multiplied by 10 by annexing one cipher, by 100 by annexing two ciphers, etc., (Art. 54,) and as division is the reverse of multiplication, cutting off one figure at the right of the dividend divides it by 10, cutting off two figures divides it by 100, etc.

The *Decimal Point* (Art. 24), by being removed as many places to the left as there are ciphers in the divisor, may be used to denote where the cutting off is made, and the remainder will be the part on the right of the point. Thus,

1326 ÷ 10 = 132.6 = $132\frac{6}{10}$, and is read, one hundred thirty-two units and six tenths; 1326 ÷ 100 = 13.26 = $13\frac{26}{100}$, and is read, thirteen units and twenty-six hundredths; 1326 ÷ 1000 = 1.326 = $1\frac{326}{1000}$, and is read, one unit and three hundred twenty-six thousandths, etc. That is,

---

When the divisor is 10, 100, 1000, etc., how can the quotient be obtained at once? How do you denote where the cutting-off is *made? Why is the* decimal point so called? *Ans.* From *decem, the Latin for ten.*

The first order at the right of the decimal point expresses tenths; the second, hundredths; the third, thousandths, etc.

26. Divide 1975 by 10. Ans. $197\frac{5}{10}$.
27. Divide 4576 by 100. Ans. $45\frac{76}{100}$.
28. Divide 901 by 100. Ans. $9\frac{1}{100}$.
29. Divide 1860 by 10. Ans. 186.
30. Divide 17930 by 1000. Ans. $17\frac{930}{1000}$.

31. At 6 dollars each, how many sheep can be purchased for 1806 dollars? Ans. 301.

32. At 10 dollars a barrel, how many barrels of flour can be purchased for 1440 dollars? Ans. 144.

33. 7 men working together earned 1547 dollars; what is each man's share of it? Ans. 221 dollars.

34. A gentleman dying left 24560 dollars, to be divided equally between his wife and three sons. What will be the share of each? Ans. 6140 dollars.

35. John Stevens sold melons at 9 cents each, to the amount of 2943 cents. How many did he sell?

36. How many watches, at 100 dollars each, can be bought for 978000 dollars?

37. If a man can do a piece of work in 365 days, how many days will it take 5 men to do the same? Ans. 73.

38. If a prize amounting to 95163 dollars be divided between 8 men, how much will each receive?
Ans. $11895\frac{3}{8}$ dollars.

39. At 6 dollars a ton, how much coal can be bought for 19201 dollars? Ans. $3200\frac{1}{6}$ tons.

What does the first order at the right of the decimal point express? The second order? The third?

LONG DIVISION.

66. Division is called **Long Division** when the work is all written.

1. Divide 34531 by 15.

OPERATION.

Dividend.

Divisor $15)34531(2302\frac{1}{15}$ *Quotient.*

30

45

45

031

30

1 *Remainder.*

We write the divisor, and begin to divide as in short division.

15 is contained in 34 thousands, 2 thousand times; we write the 2 thousand in the quotient, at the right of the dividend.

15 × 2 thousand = 30 thousand, which subtracted from 34 thousand leaves 4 thousand. Bringing down the figure of the next lower order of the dividend, we have 45 hundred, in which 15 is contained 3 hundred times; we write the 3 hundred in the quotient.

15 × 3 hundred = 45 hundred, which subtracted from 45 hundred leaves nothing. Bringing down the next figure, we have 3 tens, in which 15 is contained 0 tens times; we write the 0 tens in the quotient, and bringing down the next figure, we have 31 units, in which 15 is contained 2 times; we write the 2 in the quotient.

Multiplying and subtracting, as before, we have a remainder of 1; and, for the complete quotient, $2302\frac{1}{15}$.

In practice, we may say: 15 in 34, 2 times; write 2 in the quotient; 15 × 2 = 30, which from 34 leaves 4. Bring down 5; 15 in 45, 3 times; write 3 in the quotient; 15 × 3 = 45, which from 45 leaves 0. Bring down 3; 15 in 3, 0 times; write 0 in the quotient. Bring down 1; 15 in 31, 2 times; 15 × 2 = 30, which from 31 leaves 1. Answer, $2302\frac{1}{15}$.

When is Division called Short Division? (63.) When called *Long* Division? (66.) How do Long and Short Division differ?

Rule. — *Write the divisor at the left of the dividend.*

Begin at the left; divide the number expressed by the fewest figures of the dividend that will contain the divisor, and write the quotient at the right of the dividend.

Multiply the divisor by this quotient; subtract the product from the part of the dividend used, and to the remainder bring down the next figure of the dividend.

Divide as before, and so continue till all the figures of the dividend have been used.

If there be a final remainder write it, with the divisor beneath, after the integral part of the quotient.

PROOF. — The same as in short division.

Examples.

2. Divide 26639 by 19.

OPERATION.		PROOF.	
19)26639(1402$\frac{1}{19}$, *Ans.*		1402	
19		19	
76		12618	
76		1402	
039		26638	
38		1	*Remainder.*
1 *Remainder.*		26639	

How do you write the Divisor? Where do you begin to divide? How do you multiply and subtract? What is the Rule? The Proof?

3. Divide 88763 by 32.

OPERATION.

$$
\begin{array}{l}
32)88763(2773\tfrac{27}{32} \\
\underline{64} \\
247 \\
\underline{224} \\
236 \\
\underline{224} \\
123 \\
\underline{96} \\
27
\end{array}
$$

4. Divide 400000 by 96.

OPERATION.

$$
\begin{array}{l}
96)400000(4166\tfrac{64}{96} \\
\underline{384} \\
160 \\
\underline{96} \\
640 \\
\underline{576} \\
640 \\
\underline{576} \\
64
\end{array}
$$

Divide,

5. 6780 by 15. Ans. 452.
6. 9125 by 25.
7. 58752 by 32. Ans. 1836.
8. 40566 by 19. Ans. $2135\frac{1}{19}$.
9. 32572 by 34.
10. 29880 by 95.
11. 3762 by 24. Ans. $156\frac{18}{24}$.
12. 7168 by 28. Ans. 256.
13. 6750 by 15. Ans. 450.
14. 1748 by 18. Ans. $97\frac{2}{18}$.
15. 22200 by 222.
16. 9125 by 25. Ans. 365.
17. 45678 by 16.
18. 462442 by 183.
19. 5904 by 82. Ans. 72.
20. 208126 by 345. Ans. $603\frac{91}{345}$.
21. 65705 by 73. Ans. $900\frac{5}{73}$.
22. 14220 by 316.
23. 2780 by 13.
24. 42024 by 103. Ans. 408.
25. 27030 by 19.
26. 45817 by 35.
27. 6318 by 27. Ans. 234.
28. 77112 by 204. Ans. 378.
29. 9990 by 54.

REVIEW. — What is an Operation? (13.) A Rule? (14.) *What an* Example? (p. 20.) What is an Exercise? (p. 10.)

30. 34331 by 11.
Ans. 3121.

31. 11520 by 12.

32. 66327 by 320.
Ans. $207\frac{87}{320}$

33. 46800 by 144.

34. 241086 by 1132.
Ans. $212\frac{118}{1132}$.

35. 727604 by 3602.
Ans. 202.

36. 437850 by 75.
Ans. 5838.

37. If 630 dollars be divided equally among 15 men, how much will each receive? Ans. 42 dollars.

38. At the rate of 31 miles an hour, how long will a train of cars be in going 1116 miles? Ans. 36 hours.

39. If a ship sail 3648 miles in 24 days, at what rate is that per day? Ans. 152 miles.

40. If 12 schools together have 1932 scholars, what is the average number of each? Ans. 161 scholars.

41. When 16 acres cost 2304 dollars, what is the cost per acre? Ans. 144 dollars.

42. If 100 men share equally a prize of 17650 dollars, what is the share of each?

43. When 110 plows are worth 1320 dollars, what is each plow worth? Ans. 12 dollars.

44. If 21 bushels of wheat weigh 1260 pounds, what is the weight of one bushel?

45. 25 workmen received for a job 3155 dollars, to be equally divided among them; how much did each one receive? Ans. $126\frac{5}{25}$ dollars.

46. A field containing 11 acres produced 746 bushels of corn; what was the yield per acre?

47. If a person can copy 50 words in a minute, how long will he be in copying 47569 words?
Ans. $951\frac{19}{50}$ minutes.

REVIEW. — What is Division? (59.) What is the number to be divided called? (59.) The number by which we divide? (59.)

48. At a sale, 8 carriages sold for 6972 dollars; what was the average rate? Ans. 249 dollars.

49. The salary of the President of the United States is 50000 dollars a year; how much is that a day, there being 365 days in a year? Ans. $136\frac{360}{365}$ dollars.

50. The distance from the earth to the moon is 240000 miles; how long would a person be in reaching it, supposing he could move regularly 30 miles an hour?

Ans. 8000 hours.

REVIEW.

1. Add together ninety thousand three hundred and forty, one thousand three hundred and six, and two.

Ans. 91648.

2. Subtract five thousand three hundred and seventeen from sixteen thousand one hundred and thirty-four.

3. A and B start from the same point and travel in opposite directions. How far apart are they when A has gone 67 miles and B 139 miles?

4. The smaller of two numbers is 569, and their difference is 731; what is the larger number?

Ans. 1300.

5. A and B start from the same point and travel in the same direction. How far apart are they when A has gone 198 miles and B 319 miles?

6. The product of two factors is 4680, and one of the factors is 15; what is the other factor? Ans. 312.

7. Bought goods to the amount of 3467 dollars, paying for them in corn 421 dollars, in wheat 968 dollars, and the rest in money; how much money was paid?

Review. — What is Notation? (32.) Numeration? (36.)

8. From $(612 + 12)$ take $(316 - 32)$. Ans. 340.

9. $(591 \times 3) + 29 =$ how many? Ans. 1802.

10. Gave 3375 dollars and 5 horses, worth each 125 dollars, for a farm of 160 acres; what was the cost of the farm per acre? Ans. 25 dollars.

11. What number is that which if added to 1000 will amount to 5417? Ans. 4417.

12. Bought 3 barrels of flour at 12 dollars a barrel, and 6 tons of hay at 30 dollars a ton, and paid for the whole 54 sheep; how much each was allowed for the sheep? Ans. 4 dollars.

13. $(565 + 10) \div 25 =$ how many? Ans. 23.

14. Paid one-half of 36730 dollars for some oil-lands, which turned out to be 963 dollars more than their true value; what were they worth? Ans. 17402 dollars.

15. If the yearly income of a farmer is 2163 dollars, and from it he has to pay 122 dollars for repairs, 35 dollars for taxes, 516 dollars for hired help, and 805 dollars for support of his family, how much can he save? Ans. 685 dollars.

16. At 85 dollars per acre, how many acres of land can be bought for 8575 dollars?

Ans. 100, with 75 dollars left.

17. At 150 dollars each, how many horses can be bought for 2165 dollars?

18. A drover has 2144 dollars, with which he wishes to buy horses at 165 dollars each, and then lay out the remainder for sheep at 4 dollars each. How many of each can he buy? Ans. 12 horses, and 41 sheep.

REVIEW. — What is Addition? (39.) Subtraction? (44) Multiplication? (49.) Division? (59.)

UNITED STATES MONEY.

69. United States Money is the legal currency of the United States.

Table.

10 mills	are	1 cent,	marked c.
10 cents,		1 dime,	d.
10 dimes,		1 dollar,	$
10 dollars,		1 eagle,	E.

70. Coins are pieces of metal stamped by Government to circulate as money.

71. The coins of the United States are of bronze, nickel, silver, and gold: —

BRONZE — cent; NICKEL — three-cent and five-cent SILVER — dime, twenty-cent, quarter-dollar, half-dollar, dollar, and the trade dollar; GOLD — dollar, quarter-eagle, three dollar, half-eagle, eagle, and double-eagle.

What is United States Money? Recite the Table. What are Coins? Of what are the coins of the United States? Which are of bronze? Of nickel? Of silver? Of gold?

72. The **Dollar** is the *unit* of United States Money.

73. In accounts, eagles are expressed in dollars, dimes in cents, and mills often in parts of a cent.

Thus, 3 eagles are called 30 dollars; 2 dimes, 20 cents; and 5 mills, often, half a cent.

Eagles are *tens* of dollars, dimes are *tenths*, cents *hundredths*, and mills *thousandths*, of a dollar, and they are separated from dollars by the decimal point, (.)

Hence 44 eagles 4 dollars 4 dimes 4 cents 4 mills are written

$444.444,

and read four hundred forty-four dollars forty-four cents four mills.

74. In United States Money, 10 of a lower denomination are 1 of the next higher. Hence,

United States Money may be written, added, subtracted, multiplied, and divided, by preceding rules.

MENTAL EXERCISES.

1. How many mills are there in 7 cents?

SOLUTION. — *Since in* 1 *cent there are* 10 *mills, in* 7 *cents there are* 7 *times* 10 *mills, which are* 70 *mills. Therefore, in* 7 *cents there are* 70 *mills.*

2. How many cents are there in 8 dimes?
3. How many dollars are there in 5 eagles?
4. How many dimes are there in 6 dollars?
5. How many dollars are there in 4 half-eagles?

What is the Unit of United States Money? How are Eagles expressed in accounts? Dimes? Mills, often? What are Eagles? Dimes? Cents? Mills? How are dimes, cents, and mills separated from dollars? How many of a lower denomination in United States Money make 1 of the next higher? How may United States Money be written, added, subtracted, etc.?

6. How many cents are there in 6 half-dimes?
7. How many cents are there in 4 quarter-dollars?
8. How many dollars are there in 3 double-eagles?
9. How many cents are there in 70 mills?

SOLUTION. — *Since in* 10 *mills there is one cent, in* 70 *mills there are as many cents as* 10 *mills are contained times in* 70 *mills, which are* 7. *Therefore, there are* 7 *cents in* 70 *mills.*

10. How many dimes are there in 60 cents?
11. How many dollars are there in 100 cents?
12. How many dollars are there in 300 cents?
13. How many dollars are there in 70 dimes?
14. How many cents are there in 90 mills?
15. How many eagles are there in 110 dollars?

WRITTEN EXERCISES.

Copy and read the following: —

1. $7.65.	4. $16.125.	7. $100.06.
2. $12.20.	5. $77.005.	8. $363.55.
3. $37.02.	6. $11.111.	9. $217.755.

Write in figures: —

10. Twenty-five dollars fifteen cents. Ans. $25.15.
11. Sixty-three dollars seven cents. Ans. $63.07.
12. One hundred dollars ten cents. Ans. $100.10.
13. Two dollars two cents five mills. Ans. $2.025.
14. No dollars sixteen cents. Ans. $0.16.
15. Twenty-five dollars five mills. Ans. $25.005.
16. Five hundred seventy-nine dollars. Ans. $579.
17. Three dollars no cents eight mills.
18. One thousand dollars one cent. Ans. $1000.01.

Give the Solution of Ex. 9.

ADDITION.

75. 1. Add \$21.38, \$1.82, and \$31.015.

OPERATION.

$$\begin{array}{r} \$21.38 \\ 1.82 \\ 31.015 \\ \hline \$54.215 \end{array}$$

For convenience, we write the numbers so that figures of the same order shall stand in the same column, and begin at the right to add, observing to place the decimal point between dollars and cents.

(2.)	(3.)	(4.)	(5.)
\$100.31	\$91.636	\$167.345	\$18.45
19.435	160.05	70.00	29.68
3.005	10.109	707.07	106.05
Ans. \$122.750			\$154.18

6. Add \$960.17, \$817.05, \$3.09, and \$29.19?

7. Bought a coat for \$12.50, a vest for \$6, a pair of shoes for \$2.12½, and a handkerchief for \$0.87½; how much did the whole cost? Ans. \$21.50.

8. Sold a horse for \$208, a cow for \$70, and some hay for \$27.50; how much did the whole bring?

9. Albert Dayton owes A \$145.91, B \$1690.84, C \$345.11, D \$918.85, and E \$1144.75; what is the whole amount of his debts? Ans. \$4245.46.

10. If I give for a plow \$10.63, for a wagon \$86.75, for a load of hay \$17.15, and for a hoe \$1.25, how much do they all cost me?

11. A merchant owes three hundred seven dollars, six hundred twenty-two dollars eighty-eight cents, sixty-two dollars 17 cents 5 mills, 325 dollars 12 cents 5 mills; how much does he owe?

How are the numbers written in Addition of United States Money? Why? *Ans.* Because only numbers of the same denomination can be added? Where do you begin to add? Where do you place the decimal point in the result?

SUBTRACTION.

76. 1. Subtract \$109.02½ from \$963.03.

OPERATION.

\$963.030
109.025
\$854.005

For convenience, we write the subtrahend under the minuend, so that figures of the same order shall stand in the same column, and begin at the right to subtract, observing to place the decimal point between dollars and cents.

Here, in the subtrahend, we write the ½ cent as 5 mills; and in the minuend note the place of mills by 0.

(2.)	(3.)	(4.)	(5.)
\$807.09	\$420.35	\$87.305	\$8.340
191.63	319.06	19.32	6.765
\$615.46			\$1.575

6. Subtract \$59.23 from \$319:07. Ans. \$259.84.

7. Subtract \$87.62½ from \$93.08. Ans. \$5.455.

8. On going to the village I had in my pocket \$102.50, but I spent in a store \$37.85; how much had I then left? Ans. \$64.65.

9. From \$7 take 7 cents. Ans. \$6.93.

10. From \$11 take 1 mill. Ans. \$10.999.

11. Bought goods for \$5675.50, and sold them for \$7500.31; how much was made by the operation?

Ans. \$1824.81.

12. Bought a slate for 25 cents, an arithmetic for 60 cents, some other books for \$12.125, and some stationery for \$1.62½, and gave in payment a twenty-dollar note; how much change should be received? Ans. \$5.40.

How do you write the Numbers for Subtracting? Where do you *begin to subtract*? Where place the Decimal Point?

MULTIPLICATION.

77. 1. Multiply $316.12 by 5.

OPERATION.

$316.12
5
$1580.60

We begin at the right to multiply, observing to place the decimal point between dollars and cents in the product.

(2.)	(3.)	(4.)
$7.14	$210.06	$40.625
14	37	24
2856	147042	162500
714	63018	81250
$99.96	$7772.22	$975.000

5. Multiply $192.075 by 108. Ans. $20744.10.

6. Multiply $47.56 by 16.

7. What cost 9 tons of coal at $5.50 per ton?

8. What will be the cost of 563 bushels of wheat at $2.25 per bushel? Ans. $1266.75.

9. Bought 260 pounds of sugar at $0.125 per pound, and sold it at $0.15 per pound; what was the gain in the operation? Ans. $6.50.

10. Sold 4 cords of wood at $8 a cord, 3 tons of hay at $32.50 each, taking in part payment a cow worth $63 and the balance in money; how much money was received? Ans. $66.50.

11. What cost 36 pairs of shoes at $2.12½ each?

12. What cost 31 yards of broadcloth at $6.50 per yard? Ans. $201.50.

REVIEW. — What is United States Money? What are Coins? Of what metals are the coins of the United States?

13. Sold 316 pairs of boots at $6.50 per pair, and received in payment $1000 down, and the balance in a note. For how much was the note? Ans. $1054.

DIVISION.

78. 1. Divide $65.56 by 8.

OPERATION.

$8)\$65.56$

$\$8.19\frac{4}{8}$

Or,

$8)\$65.560$

$\$8.195$

Dividing the dollars and the cents, we have a remainder, which we write as a part of a cent.

In the second operation, the division is extended to mills by supplying the mills' place in the dividend. The two answers, although a little different in form, express the same value.

2. Divide $497 by 4.

OPERATION.

$4)\$497$ $\$124\frac{1}{4}$ *Or,* $4)\$497.00$ $\$124.25$

(3.) $9)\$648.81$ — 72.09

(4.) $8)\$108$

(5.) $6)\$986.40$

(6.) $7)\$684.614$ — $\$97.802$

7. Divide $2764.25 by 7. Ans. $394.892+.

Here, in the answer, the sign + shows that the division is not exact.

How may the division be extended from dollars to cents, when there is a remainder, and no cents in the dividend? *Ans.* By supplying the two places of cents by ciphers. How may the division be extended to mills from cents, when there is a remainder, and no mills in the dividend? *Ans.* By supplying the place of mills by a cipher. What does the sign + written after an answer indicate? Why do the two answers obtained to Ex. 2 express the same value?

8. Paid $78 for 12 sheep; how much was that for each? Ans. $6.50.

9. A merchant paid $255 for 75 sheep; how much was that a head? Ans. $3.40.

10. If $11.44 be paid for 26 yards of drilling, what is the cost per yard? Ans. $.44.

11. If 24 men receive $637 to be equally divided among them, how much is each man's share?

Ans. $26.541+.

12. Bought 25 yards of cambric for $4; how much was that a yard? Ans. $.16.

13. If a merchant paid $240 for 32 tons of coal, what was that a ton?

14. A farmer expended $427.12½ for 201 sheep; for how much apiece must he sell them, so as neither to gain nor lose? Ans. $2.125.

BILLS AND ACCOUNTS.

79. A **Bill** is a written statement of merchandise bought or sold, or of services rendered.

80. An **Account** is a written statement of debt and credit between two parties.

The party owing is the *Debtor*, and the party owed is the *Creditor*.

81. A bill is *receipted* when its payment is acknowledged by the party in whose favor it is, or by some one authorized to act for him.

82. In the settlement of an account it is required to find the difference between the items of debt and the items of credit, or the balance.

What is a Bill? An Account? Which party is the Debtor? Which the Creditor? When is a bill receipted?

Find the amount due on each of the following bills, or accounts.

(1.) New York, *May 16, 1871.*

Mr. James Walsh

Bought of J. B. DALTON & CO.,

			$
16 reams Paper,	*at*	*$7.25,*	
25 Slates,	*"*	*.40,*	
20 Inkstands,	*"*	*.37,*	
			$133.40

Received payment,

J. B. Dalton & Co.

(2.) Philadelphia, *July 5, 1871.*

Burgess & Co.

Bought of WILLIAM HOLSTEIN,

			$
160 bu. Peaches,	*at*	*$2.50,*	
58 " Peaches,	*"*	*2.25,*	
30 " Apples,	*"*	*1.25,*	
20 " Apples,	*"*	*.90,*	
			$586.00

Received Payment,

(2.) New Orleans, *Sept. 9, 1871.*

Mr. Louis Barrot,

To FELIX DUMAS & CO., Dr.

To 630 gal. Molasses, at $.60,
" 1763 lbs. Sugar " .17,
" 3610 " Cotton, " .31,

$1330.81

(3.) St. Louis, *July 1, 1871.*

Mr. John W. Curtis,

1871. **To HENRY D. FELT, Dr.**

Jan. 3. To 365 bu. Wheat, at $2.20,
Feb. 7. " 160 " Corn, " .50,
June 9. " 192 " Barley, " .60,
" 12. " 20 bbl. Flour, " 3.50,

$1163.20

1871. **Cr.**

March 3. By Cash, $303.00
May 14. " Merchandise, 175.00
June 15. " 50 bags, at $.60, 30.00

$1003.00

Balance due H. D. F., $160.20

Received payment,

Henry D. Felt.

FACTORING.

83. 1. How many times 2 is 10? What two whole numbers multiplied together will produce 10?

2. What two whole numbers multiplied together will produce 6? Will produce 15?

3. What two whole numbers greater than 1, multiplied together will produce 21? What three whole numbers greater than 1, multiplied together will produce 30?

4. What whole number taken as a factor twice, will produce 4? Will produce 9?

5. Name some whole number that cannot be produced by multiplying together two whole numbers greater than 1.

6. Name some number that is the product of two or more integers greater than 1.

7. Name some whole numbers that can be divided by 2 without a remainder. That can be divided by 2 and by 3 without a remainder.

8. Name some whole number that can only be divided by itself or by 1, without a remainder.

84. An **Integer, or Integral Number,** is a whole number.

85. A **Prime Number** is an integer which cannot be divided without a remainder by any integer other than itself, or 1; as 1, 2, 3, 5, 7.

86. A **Composite Number** is an integer which can be divided without a remainder by some integer other than itself, or 1; as 4, 6, 8, 8.

What is an Integer? A Prime Number? A Composite Number?

87. A **Factor** of a number is any integer contained in it without a remainder; thus, 2 is a factor of 4.

A **Prime Factor** is a prime number.

88. **Factoring** is the process of finding the factors of a composite number.

A PRINCIPLE OF FACTORING.

89. *A number is equal to the product of its prime factors.*

WRITTEN EXERCISES.

90. 1. Find the prime factors of 84.

OPERATION.

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

By trial, we find that 84 is composed of two factors, 2 and 42; of which 2 is prime, and 42 is composite.

The 42 we find composed of the factors 2 and 21; of which 2 is prime, and 21 is composite.

The 21 is composed of two factors, 3 and 7, both prime.

Therefore, the prime factors required are 2, 2, 3, and 7.

Rule. — *Divide the given number by any prime number greater than 1, that will divide it without a remainder, and the quotient, if composite, in the same manner; and thus continue until the quotient is prime. The divisors and the last quotient will be the prime factors required.*

PROOF. — The product of the prime factors will equal the given number, if the work is right.

What is a Factor? A Prime Factor? What is Factoring? How do you divide the given number? If the quotient is composite, how do you proceed? How long do you continue? What will be the prime factors? Recite the Rule. What is the Proof?

Examples.

2. What are the prime factors of 44? Ans. 2, 2, 11.
3. What are the prime factors of 57?
4. What are the prime factors of 68? Ans. 2, 2, 17.
5. What are the prime factors of 70? Ans. 2, 5, 7.
6. What are the prime factors of 95?
7. What are the prime factors of 210?

Ans. 2, 3, 5, 7.

8. What are the prime factors of 735?

Ans. 3, 5, 7, 7.

MULTIPLICATION BY FACTORS.

91. 1. Let it be required to multiply 452 by 63.

OPERATION.

$$\begin{array}{r} 452 \\ 7 \\ \hline 3164 \\ 9 \\ \hline \text{Ans. } 28476 \end{array}$$

63 is equal to 7 times 9; hence 63 times 452 is equal to 9 times 7 times 452. 7 times 452 gives 3164, and 9 times 7 times 452, or 9 times 3164, gives 28476.

Therefore, 452 multiplied by 63 is equal to 28476.

Rule.—*Multiply the multiplicand by one factor, and this product by another, and so on, until all the factors have been used; the last product will be the one required.*

Examples.

2. Multiply 365 by 35, using its factors.

When the multiplier is a composite number, how can you multiply? *Ans.* By its factors. ° What is the Rule?

3. Multiply 205 by 27, or 3 × 9.
4. Multiply 265 by 32, or 4 × 8. Ans. 8480.
5. Multiply 521 by 16, or 4 × 4. Ans. 8336.
6. Multiply 1124 by 60, or 10 × 6. Ans. 67440.

Here, we annex a cipher to the multiplicand for the product of it by 10 (Art. 54), and then multiply by the 6; that is, *we annex the cipher to the multiplicand, and multiply by the number expressed by the significant figure.*

7. Multiply 301 by 50, or 10 × 5. Ans. 15050.
8. What cost 42 horses, at $165 each?
9. Multiply 2400 dollars by 15, using its factors.

Ans. 36000 dollars.

10. Required the weight of 56 bags of grain, each weighing 112 pounds. Ans. 6272 pounds.
11. Required the cost of 249 bales of goods, at $2 per bale.
12. A farm has been divided into 49 lots, each containing 570 square rods; how large was the farm?

Ans. 27930 square rods.

CANCELLATION.

92. **Cancellation** is the process of shortening operations in division, by rejecting equal factors from the dividend and divisor.

Cancellation depends upon the principle that

Dividing both the dividend and divisor by the same number will not change the quotient.

What is Cancellation? Upon what principle does Cancellation depend?

To prove this principle, let the dividend be 16 and the divisor 4. Then, $16 \div 4 = 4$. Now, if we divide the dividend and divisor by some number, as 2, the dividend becomes 8, and the divisor, 2; and $8 \div 2 = 4$, the same result as before.

93. 1. Divide 60 by 15.

OPERATION.

$$\frac{60}{15} = \frac{\not{3} \times 20}{\not{3} \times 5} = 4, \textit{Ans.}$$

For convenience, we indicate the division, by writing the dividend over the divisor.

Factoring, the dividend becomes 3×20, and the divisor 3×5.

Canceling, by crossing the factor 3, common to both dividend and divisor, we have $\frac{20}{5}$, or $20 \div 5$, equal to 4, the quotient required.

2. Divide 2 times 45 by 9.

OPERATION.

$$\frac{45 \times 2}{9} = \frac{\not{9} \times 5 \times 2}{\not{9}} = 10, \textit{Ans.}$$

Indicating the division, and factoring, the dividend becomes $9 \times 5 \times 2$.

Canceling the factor 9, common to both dividend and divisor, we have left in the dividend 5×2, or 10, which is the quotient.

3. Divide 16 times 21 by 8 times 14.

OPERATION.

$$\frac{\not{2} \quad 3}{\not{16} \times \not{21}} = 3, \textit{Ans.}$$
$$\not{8} \times \not{14}$$
$$\not{7}$$

Canceling the factor 8, common to both divisor and dividend, we have left in the dividend 2, in place of 16.

Canceling the factor 2, common to both divisor and dividend, we have left in the divisor 7, in place of 14.

Canceling the factor 7, common to

Prove this principle.

the dividend and divisor, we have left in the dividend 3, which is the quotient.

Rule. — *Cancel all the factors common to both dividend and divisor, and then divide the product of the remaining factors of the dividend by the product of the remaining factors in the divisor.*

When a factor is canceled, 1 is understood to remain, but does not require to be written, except in the quotient, when there are no other factors.

Examples.

4. Divide the product of 16×14 by 8×7.

Ans. 4.

5. Divide the product of $21 \times 15 \times 5$ by $18 \times 10 \times 3$. Ans. $2\frac{1}{12}$.

6. Divide the product of $33 \times 35 \times 6$ by $11 \times 7 \times 3$.

7. Divide the product of 72×8 by 12×6.

Ans. 8.

8. How many bushels of potatoes, at 60 cents a bushel, must be given in exchange for 4 calico dresses, each containing 12 yards, at 30 cents a yard?

9. How many bushels of peaches, at $1.60 a bushel, must be given for 5 bushels of apples, at $1.28 a bushel?

Ans. 4 bushels.

10. Exchanged 3 firkins of butter, each containing 56 pounds, at 24 cents a pound, for 6 bags of coffee, each containing 32 pounds; how much was the coffee a pound?

Ans. 21 cents.

°What is the Rule? When a factor is canceled, what remains? When is the factor 1 to be written?

ANALYSIS.

94. Analysis, in Arithmetic, is the process of solving a question or example by considering the relation of its numbers.

No rules, therefore, are required for the solution of questions by analysis.

MENTAL EXERCISES.

1. If 6 oranges cost 18 cents, what will 8 oranges cost?

SOLUTION. — *If* 6 *oranges cost* 18 *cents*, 1 *orange will cost* 1 *sixth of* 18 *cents, which is* 3 *cents, and* 8 *oranges will cost* 8 *times* 3 *cents, which are* 24 *cents. Therefore, if* 6 *oranges cost* 18 *cents,* 8 *oranges will cost* 24 *cents.*

2. If 5 peaches cost 10 cents, how much will 10 peaches cost?

3. If 7 yards of velvet cost $28, how much will 9 yards cost?

4. If 8 pounds of sugar cost 72 cents, how much will 10 pounds cost?

5. When 36 cents are paid for 4 quarts of berries, how much must be paid for 7 quarts?

6. What will 10 quarts of milk cost, if 5 quarts cost 40 cents?

7. When $6 will buy 24 yards of cloth, how many yards will $8 buy?

8. If 9 pounds of veal cost 54 cents, how many pounds can be bought for 42 cents?

9. If 7 plows can be bought for $77, how many can be bought for $44?

10. When $8 will buy 40 rakes, how many can be bought for $11?

11. If 12 men can do a piece of work in 6 days, how many days will it take 9 men to do the same?

SOLUTION. — *If* 12 ***men*** *can do the work in* 6 *days, it will take* 1 ***man*** 12 *times* 6 *days, or* 72 *days, to do it, and* 9 ***men*** 1 *ninth of* 72 *days, or* 8 *days.*

12. If 7 men can dig a ditch in 8 days, how many days will be required for 14 men to dig it?

13. If 6 horses eat a bag of oats in 10 days, how many horses will eat the same quantity in 15 days?

14. How long will it take 5 men to reap a field, if it take 15 men 3 days to reap it?

15. How long will it take 9 men to build a wall, if it take 6 men 6 days to build it?

16. If 7 books cost $21, what will 11 books cost?

17. When 8 barrels of flour will pay for 12 yards of broadcloth, at $6 a yard, how much is the flour per barrel?

18. If 5 men can hoe a field of corn in 9 days, in what time can 15 men hoe it?

19. When 10 oranges will pay for 20 apples, at 2 cents each, what is the price of each orange?

20. When 10 bushels of wheat will pay for 5 cords of wood, at 4 dollars per cord, how much is the wheat per bushel?

21. If 9 pencils cost 54 cents, what will 12 pencils cost, at the same rate?

REVIEW. — What is an Integer? (84.) A Prime Number? (85.) A Composite Number? (86.) A Factor? (87.)

WRITTEN EXERCISES.

1. If 9 acres of land produce 315 bushels of wheat, how many bushels will 7 acres produce?

OPERATION.

$$\frac{35}{\cancel{9}\,\cancel{\$315} \times 7}{\cancel{\$}} = 245 \text{ bu., } \textit{Ans.}$$

If 9 acres of land produce 315 bushels of wheat, 1 acre will produce 1 ninth of 215 bushels, or 35 bushels, and 7 acres will produce 7 times 35 bushels, or 245 bushels.

2. If 10 yards of cloth cost $15, what will 8 yards cost? Ans. $12.

3. If 5 pounds of cheese cost 45 cents, what will 18 pounds cost? Ans. $1.62.

4. If 26 yards of cloth cost $78, what will 17 yards cost?

5. If 72 bushels of apples cost $36, what will 84 bushels cost? Ans. $42.

6. If 8 bushels of wheat cost 12 dollars, what will 20 bushels cost? Ans. $30.

7. If a man can earn in 5 months $176.25, how much can he earn in 12 months, or 1 year? Ans. $423.

8. When 3 books cost $1.14, what will 3 dozen cost? Ans. $13.68.

9. If 3 dozen books cost $13.68, what will 3 books cost?

10. When $12.75 will purchase 17 yards of cloth, how many yards will $2.25 purchase?

11. How far can a man travel in 7 days, at the rate of 244 miles in 4 days? Ans. 427 miles.

REVIEW. — What is a Prime Factor? (87.) Factoring? (88.) What is Cancellation? (92.) Upon what principle does it depend? (92.)

12. If 14 horses eat a load of hay in 14 days, how many horses would be required to eat the hay in 7 weeks?

Ans. 4 horses.

13. Bought 72 yards of broadcloth for 160 bushels of wheat, at $1.80 per bushel; how much was the broadcloth a yard? Ans. $4.

14. If I give $1887 for 148 barrels of flour, how many barrels can I buy for $63.75? Ans. 5 barrels.

15. If 7 men can build a house in 84 days, how many men could build it in 21 days? Ans. 28 men.

16. When $30 is paid for 25 pounds of tea, how much must be paid for 40 pounds?

17. A drover exchanged 25 horses for 550 sheep, at $5.50 each; how much did he get for each of his horses?

Ans. $121.

18. If 5 tons of hay cost $106.25, what will 13 tons cost? Ans. $276.25.

19. In what time will 48 men do a piece of work that 12 men can do in 24 days? Ans. 6 days.

20. When 15 pounds of sugar can be bought for $1.80, what will 18 pounds cost?

21. How far can a man travel in 18 days, at the rate of 378 miles in 6 days? Ans. 1134 miles.

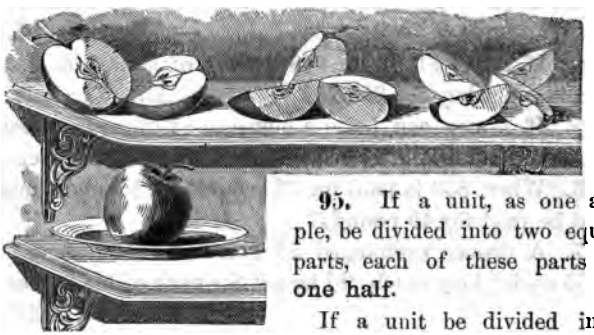
22. If a man travel 1134 miles in 18 days, how many miles will he travel in 6 days?

23. How many pounds of coffee can be bought for $21, if 30 pounds cost $6? Ans. 105 pounds.

24. When 18 pounds of sugar can be bought for $2.16, what will 15 pounds cost? Ans. $1.80.

REVIEW. — What is Analysis in Arithmetic? (94.) What is Arithmetic? (15.) Addition? (39.) Subtraction? (44.) Multiplication? (49.) Division? (59.)

COMMON FRACTIONS.



95. If a unit, as one apple, be divided into two equal parts, each of these parts is **one half.**

If a unit be divided into three equal parts, one of these parts is **one third**; two of them, **two thirds,** etc.

If a unit be divided into four equal parts, one of these parts is *one fourth;* two of them, *two fourths;* three of them, *three fourths;* etc.

In like manner, if a unit be divided into five equal parts, the parts are *fifths;* if into six equal parts, *sixths;* and so on.

96. **A Fraction** is one or more equal parts of a unit. Thus, one half, two thirds, are fractions.

97. The **Denominator** of a fraction is the number which shows into how many equal parts the unit is divided. Thus,

Three is the denominator of two thirds.

If a unit be divided into two equal parts, what is one of the equal parts called? Into three equal parts? Into four? Into five? What *is a Fraction? The* Denominator?

98. The **Numerator** of a fraction is the number which shows how many of the equal parts of the unit are taken. Thus,

Two is the numerator of two thirds.

99. A **Common Fraction** is a fraction expressed by writing the numerator above, and the denominator below a dividing line; as,

One half, written	$\frac{1}{2}$	Two fifths, written	$\frac{2}{5}$
One third, "	$\frac{1}{3}$	Three fifths, "	$\frac{3}{5}$
Two thirds, "	$\frac{2}{3}$	Four fifths, "	$\frac{4}{5}$
One fourth, "	$\frac{1}{4}$	One seventh, "	$\frac{1}{7}$
Two fourths, "	$\frac{2}{4}$	Three eighths, "	$\frac{3}{8}$
Three fourths, written	$\frac{3}{4}$	Five ninths, "	$\frac{5}{9}$
One fifth, "	$\frac{1}{5}$	Seven tenths, "	$\frac{7}{10}$

100. The **Terms** of a fraction are the numerator and denominator.

The **Unit** of a fraction is the unit, or single thing, divided into equal parts.

A **Fractional Unit** is one of the equal parts into which the unit of a fraction is divided.

101. An **Integer** may be expressed in a fractional form, by writing 1 under it, for a denominator. Thus,

2 may be written	$\frac{2}{1}$, and read,	2 ones.
3 may be written	$\frac{3}{1}$, and read,	3 ones.
7 may be written	$\frac{7}{1}$, and read,	7 ones.
10 may be written	$\frac{10}{1}$, and read,	10 ones.

What is the Numerator of a fraction? What is a Common Fraction? What are the Terms of a fraction? What is the Unit of a fraction? A Fractional Unit? How may an Integer be expressed in a fractional form?

102. A **Proper Fraction** is one whose numerator is less than the denominator; as $\frac{3}{7}$, $\frac{7}{8}$.

103. An **Improper Fraction** is one whose numerator equals or exceeds the denominator; as $\frac{5}{5}$, $\frac{8}{8}$.

104. A **Mixed Number** is a whole number with a fraction; as $3\frac{1}{4}$, read, three and one fourth.

EXERCISES.

Express by figures: —

1. Two sixths. Ans. $\frac{2}{6}$
2. Three sevenths.
3. Five eighths.
4. Six ninths.
5. Two tenths.
6. Seven elevenths.
7. Eight twelfths.
8. Nine sixteenths.
9. Seven ones.
10. Four ninths.
11. Eight seventeenths.
12. Fifteen twentieths.
13. Ten thirtieths.
14. Six ninths.
15. Eleven ones.
16. Seventeen fortieths.

Write and read the following: —

17. $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{11}$, $\frac{8}{15}$.
18. $\frac{10}{11}$, $\frac{16}{40}$, $\frac{19}{21}$, $\frac{13}{15}$.
19. $\frac{17}{10}$, $\frac{11}{13}$, $\frac{9}{4}$, $\frac{3}{2}$.
20. $\frac{39}{16}$, $\frac{17}{1}$, $\frac{14}{5}$, $\frac{6}{75}$.
21. $\frac{31}{1}$, $\frac{60}{71}$, $\frac{3}{29}$, $\frac{5}{11}$.
22. $\frac{6}{19}$, $\frac{7}{21}$, $\frac{11}{91}$, $\frac{14}{1}$.

Write the following, and tell the kind of fraction: —

23. $\frac{2}{5}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{19}{10}$.
24. $\frac{11}{12}$, $\frac{3}{11}$, $\frac{8}{13}$, $1\frac{1}{3}$.
25. $\frac{7}{9}$, $13\frac{1}{2}$, $\frac{15}{16}$, $\frac{17}{1}$.
26. $\frac{11}{15}$, $\frac{17}{13}$, $4\frac{5}{8}$, $8\frac{1}{4}$.
27. $14\frac{1}{2}$, $1\frac{13}{4}$, $\frac{5}{2}$, $11\frac{6}{7}$.
28. $17\frac{3}{4}$, $\frac{11}{3}$, $61\frac{3}{4}$, $\frac{21}{29}$.

What is a Proper Fraction? An Improper Fraction? A Mixed Number? What does a fraction indicate?

PRINCIPLES OF FRACTIONS.

105. 1. *The* **Value** *of a fraction is the quotient of the numerator divided by the denominator.*

For a fraction indicates division, the numerator answering to the dividend, and the denominator to the divisor. Thus, the quotient of 1 divided by 2 is $\frac{1}{2}$; 3 divided by 4 is $\frac{3}{4}$; and $\frac{4}{2}$ is 2.

2. *Dividing both terms of a fraction by the same number does not change its value.*

For it has been shown, that canceling equal factors in the dividend and divisor does not change its quotient; and it also follows, that

3. *Multiplying both terms of a fraction by the same number does not change its value.*

Thus, $\frac{4}{2}$, or $\frac{4 \div 2}{2 \div 2} = 2$, and $\frac{4}{2}$, or $\frac{4 \times 2}{2 \times 2} = 2$.

MENTAL EXERCISES.

1. How much is $\frac{1}{2}$ of 6? Of 8? Of 12?
2. How much is $\frac{1}{3}$ of 9? Of 12? Of 18?
3. How much is $\frac{1}{4}$ of 8? Of 12? Of 20?
4. How much is $\frac{1}{5}$ of 5? Of 15? Of 25?
5. How much is $\frac{1}{7}$ of 7? Of 14? Of 35?
6. How much is $\frac{1}{6}$ of 12? Of 24? Of 48?
7. How much is $\frac{1}{10}$ of 20? Of 70? Of 100?
8. How much is $\frac{1}{12}$ of 24? Of 36? Of 72?
9. How much is $\frac{1}{6}$ of 30? $\frac{1}{8}$ of 64? $\frac{1}{11}$ of 77?
10. How much is $\frac{1}{9}$ of 27? $\frac{1}{15}$ of 30? $\frac{1}{20}$ of 60?

What is the Value of a fraction? What effect has dividing both terms of a fraction by the same number? What also follows?

11. How much is $\frac{2}{3}$ of 30?

SOLUTION. — *Since* 1 *third of* 30 *is* 10, 2 *thirds of* 30 *is* 2 *times* 10, *or* 20. *Therefore,* $\frac{2}{3}$ *of* 30 *is* 20.

12. How much is $\frac{2}{5}$ of 15? Of 25? Of 45?
13. How much is $\frac{2}{8}$ of 36? Of 45? Of 27?
14. How much is $\frac{3}{4}$ of 40? Of 12? Of 28?
15. How much is $\frac{3}{7}$ of 21? Of 49? Of 42?
16. How much is $\frac{5}{6}$ of 18? Of 30? Of 48?
17. How much is $\frac{3}{8}$ of 24? Of 32? Of 64?
18. How much is $\frac{4}{5}$ of 20? Of 40? Of 50?
19. How much is $\frac{5}{8}$ of 32? Of 40? Of 56?
20. If a pound of sugar cost 15 cents, how much will 1 third of a pound cost? 2 thirds?
21. Paid $60 for a cow, and 2 twelfths as much for some hay; how much did the hay cost?
22. What part of 7 is 2?

SOLUTION. — *Since* 1 *is* 1 *seventh of* 7, 2 *is* 2 *times* 1 *seventh of* 7, *or* 2 *sevenths of* 7. *Therefore,* 2 *is* $\frac{2}{7}$ *of* 7.

23. What part of 5 is 3? 4? 2?
24. What part of 8 is 3? 5? 7?
25. What part of 9 is 2? 7? 8?
26. What part of 10 is 9? 3? 7?
27. What part of 16 yards is 5 yards? 11 yards?
28. What part of $20 is $7? $13? $17?
29. What part of 10 pounds is 5 pounds? 8 pounds?
30. What part of 27 tons is 3 tons? 9 tons?
31. What part of $32 is $12? $4? $16?
32. What part of 40 bushels is 20 bushels?
33. What part of 36 hogsheads is 9 hogsheads?

Explain the solution of example 11. Of example 22.

REDUCTION.

106. Reduction of Fractions is the process of changing their form, without changing their value.

Case I.

A Fraction to its Lowest Terms.

107. A fraction is in its **Lowest or Smallest Terms**, when the numerator and denominator have no common factor greater than 1.

1. Reduce $\frac{4}{10}$ to its lowest terms.

OPERATION.

$$\frac{4}{10} = \frac{2 \times \not{2}}{5 \times \not{2}} = \frac{2}{5}, \textit{Ans.}$$

Dividing both terms of a fraction by the same number, or canceling equal factors in both (Art. 107), will not change the value expressed. Now the only factor greater than 1, common to both terms of the given fraction, is 2, and canceling, we have $\frac{2}{5}$. Therefore, $\frac{4}{10}$ in its lowest terms is $\frac{2}{5}$.

Rule. — *Cancel in the numerator and denominator all the factors common to both.*

Examples.

Reduce to their lowest terms: —

2. $\frac{9}{12}$.	Ans. $\frac{3}{4}$.	7. $\frac{182}{196}$.	Ans. $\frac{13}{14}$.
3. $\frac{77}{84}$.	Ans. $\frac{11}{12}$.	8. $\frac{60}{125}$.	Ans. $\frac{12}{25}$.
4. $\frac{49}{343}$.	Ans. $\frac{1}{7}$.	9. $\frac{72}{94}$.	Ans. $\frac{36}{47}$.
5. $\frac{132}{264}$.		10. $\frac{253}{308}$.	
6. $\frac{147}{168}$.		11. $\frac{840}{1120}$.	

What is Reduction of fractions? When is a fraction in its lowest terms? Why is not the value of a fraction changed by dividing both terms by the same number? What is the Rule?

Case II.

An Integer to a Fraction with a given Denominator.

108. 1. Reduce 8 to a fraction whose denominator is 7.

OPERATION.

$\frac{8 \times 7}{7} = \frac{56}{7}$, *Ans.*

Since in 1 there are 7 sevenths, in 8 there are 8 times 7 sevenths, or $\frac{56}{7}$. Therefore, 8 is equal to $\frac{56}{7}$.

Rule.—*Multiply the whole number by the given denominator, and write the product over the denominator.*

Examples.

Reduce

2. 14 to fourths.
3. 19 to fifths.
4. 31 to eighths.
5. 98 to thirds.
6. 70 to thirty-fourths.
7. 135 to seventeenths.

8. 9 is equal to what fraction having 15 for a denominator? Ans. $\frac{135}{15}$.

9. Reduce 131 to a fraction whose denominator shall be 11. Ans. $\frac{1441}{11}$.

Case III.

A Mixed Number to an Improper Fraction.

109. 1. Reduce $5\frac{2}{3}$ to thirds.

OPERATION.

$5\frac{2}{3} = \frac{5 \times 3}{3} + \frac{2}{3} = \frac{15 + 2}{3} = \frac{17}{3}$, *Ans.*

Since in 1 there are 3 thirds, in 5 there are 5 times 3 thirds; and 5 times 3 thirds plus $\frac{2}{3}$, or 15 thirds plus 2 thirds, are 17 thirds, or $\frac{17}{3}$. Therefore, $5\frac{2}{3}$ is equal to $\frac{17}{3}$.

Explain the operation of reducing an integer to a fraction with a given denominator. Of reducing a mixed number to an improper fraction.

Rule. — *Multiply the integral part of the mixed number by the denominator of the fractional part; to the product add the numerator; and write the result over the denominator.*

Examples.

Reduce to an improper fraction: —

2. $7\frac{4}{9}$.	Ans. $\frac{67}{9}$.	6. $12\frac{15}{17}$.	Ans. $\frac{219}{17}$.
3. $16\frac{3}{7}$.	Ans. $\frac{115}{7}$.	7. $19\frac{2}{18}$.	Ans. $\frac{354}{18}$.
4. $9\frac{2}{11}$.	Ans. $\frac{101}{11}$.	8. $14\frac{3}{10}$.	Ans. $\frac{143}{10}$.
5. $6\frac{4}{13}$.		9. $161\frac{2}{40}$.	

10. In $10\frac{1}{4}$ dollars, how many fourths of a dollar?

11. In 221 bushels, how many elevenths of a bushel?

Case IV.

An Improper Fraction to an Integer or Mixed Number.

110. 1. Reduce $\frac{17}{3}$ to an integer or mixed number.

OPERATION.

$\frac{17}{3} = 5\frac{2}{3}$, *Ans.*

Since there are 3 thirds in 1 unit, there will be in 17 thirds as many units as 3 is contained times in 17, or $5\frac{2}{3}$. Therefore, $\frac{17}{3}$ is equal to $5\frac{2}{3}$.

Rule. — *Divide the numerator by the denominator.*

Examples.

Reduce to a whole or mixed number: —

2. $\frac{67}{9}$.	Ans. $7\frac{4}{9}$.	6. $\frac{108}{4}$.	Ans. 27.
3. $\frac{101}{7}$.	Ans. $14\frac{3}{7}$.	7. $\frac{405}{9}$.	Ans. 45.
4. $\frac{313}{13}$.		8. $\frac{106}{106}$.	
5. $\frac{405}{9}$.		9. $\frac{245}{8}$.	

Explain the operation of reducing an improper fraction to an integer or mixed number. Recite the Rule.

10. In $\frac{10}{3}$ of a dollar how many dollars?

Ans. $3\frac{1}{3}$.

11. What is the value of $\frac{82}{16}\frac{24}{}$ miles?

Ans. 514 miles.

COMMON DENOMINATOR.

111. Fractions have a **Common Denominator** when they have the same number for a denominator.

1. Reduce $\frac{2}{5}$ and $\frac{3}{7}$ to fractions having a common denominator.

OPERATION.

$$\left.\begin{aligned} \frac{2}{5} &= \frac{2\times 7}{5\times 7} = \frac{14}{35} \\ \frac{3}{7} &= \frac{3\times 5}{7\times 5} = \frac{15}{35} \end{aligned}\right\} \text{Ans.}$$

Since multiplying both the numerator and denominator of a fraction by the same number does not change its value (Art. 105), we multiply both terms of the fraction $\frac{2}{5}$ by 7, the denominator of the second fraction, and have $\frac{2}{5} = \frac{14}{35}$; and both terms of the fraction $\frac{3}{7}$ by 5, the denominator of the first fraction, and have $\frac{3}{7} = \frac{15}{35}$. Therefore, $\frac{2}{5}$ and $\frac{3}{7}$ are equal to $\frac{14}{35}$ and $\frac{15}{35}$.

2. Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{3}{12}$ to fractions having a common denominator.

OPERATION.

$$\left.\begin{aligned} \frac{2}{3} &= \frac{2\times 4}{3\times 4} = \frac{8}{12} \\ \frac{5}{6} &= \frac{5\times 2}{6\times 2} = \frac{10}{12} \\ & \qquad \frac{3}{12} \end{aligned}\right\} \text{Ans.}$$

Here, by multiplying both terms of $\frac{2}{3}$ by 4, and of $\frac{5}{6}$ by 2, we have $\frac{8}{12}$ and $\frac{10}{12}$, which have the same number for a denominator as the $\frac{8}{12}$. Therefore, the required fractions are $\frac{8}{12}$, $\frac{10}{12}$, and $\frac{8}{12}$.

When have fractions a Common Denominator? Explain the *manner* of finding the common denominator by the first operation.

Rule.—*Multiply both terms of each fraction by the denominators of the other fractions. Or,*

Multiply both terms of one or more of the fractions by such a number as will make all the denominators alike.

If there are integers or mixed numbers with the given fractions, they must first be reduced to improper fractions.

Examples.

Reduce to equivalent fractions having a common denominator: —

3. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Ans. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.
4. $\frac{1}{8}$, $\frac{2}{3}$, and $\frac{1}{24}$. Ans. $\frac{3}{24}$, $\frac{16}{24}$, $\frac{1}{24}$.
5. $\frac{2}{5}$, 4, and $\frac{3}{10}$. Ans. $\frac{4}{10}$, $\frac{40}{10}$, $\frac{3}{10}$.
6. $\frac{3}{4}$, $\frac{5}{9}$, and $\frac{7}{12}$. Ans. $\frac{27}{36}$, $\frac{20}{36}$, $\frac{21}{36}$.
7. $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{4}{15}$.
8. $\frac{3}{8}$, $\frac{2}{16}$, and $\frac{1}{4}$.
9. $\frac{5}{6}$ and $\frac{3}{8}$.
10. $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{3}{2}$. Ans. $\frac{20}{8}$, $\frac{24}{8}$, $\frac{10}{8}$, $\frac{3}{8}$.
11. $2\frac{1}{2}$, 3, $\frac{5}{4}$, and $\frac{3}{8}$.

ADDITION.

112. Fractions may be added by means of a common denominator. Thus,

We can add *halves* and *halves*, *thirds* and *thirds*, *fourths* and *fourths*, etc.; but we can not add directly *halves* and *thirds*, *thirds* and *fourths*, etc., any more than we can add other things of different kinds, as dollars and days.

1. Find the sum of $\frac{1}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.

OPERATION.

$$\frac{1}{5}+\frac{3}{5}+\frac{4}{5}=\frac{1+3+4}{5}=\frac{8}{5}=1\tfrac{3}{5},\ \textit{Ans.}$$

Recite the Rule. By what means may fractions be added?

The sum of 1 *fifth*, 3 *fifths*, and 4 *fifths*, is 8 *fifths*, or $\frac{8}{5}$ which, reduced, gives $1\frac{3}{5}$.

2. Find the sum of $\frac{3}{4}$ and $\frac{4}{5}$.

OPERATION.

$$\frac{3}{4}+\frac{4}{5}=\frac{15}{20}+\frac{16}{20}=\frac{15+16}{20}=\frac{31}{20}=1\frac{11}{20}, \textit{Ans.}$$

Since numbers must be of the same name or kind, in order to be added, we reduce the given fractions to equivalent fractions having a common denominator, and have 15 *twentieths*, and 16 *twentieths*, which, added, give 31 *twentieths*, $\frac{31}{20}$, or, by reduction, $1\frac{11}{20}$. Therefore, the sum of $\frac{3}{4}$ and $\frac{4}{5}$ is $1\frac{11}{20}$.

Rule.—*Reduce the fractions, if necessary, to equivalent fractions having a common denominator, and write the sum of the numerators over the common denominator.*

Examples.

Find the sum of

3. $\frac{4}{9}$, $\frac{7}{9}$, and $\frac{11}{9}$. Ans. $2\frac{4}{9}$.
4. $\frac{8}{15}$ and $\frac{8}{20}$. Ans. $\frac{56}{60}$.
5. $\frac{3}{5}$ and $\frac{4}{7}$. Ans. $1\frac{6}{35}$.
6. $\frac{4}{7}$, $\frac{2}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$.
7. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{7}{8}$, and $\frac{1}{4}$.
8. $\frac{1}{8}$, $\frac{3}{10}$, and $\frac{4}{12}$. Ans. $\frac{91}{120}$.
9. $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{5}$. Ans. $2\frac{111}{180}$.
10. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{7}$. Ans. $2\frac{11}{84}$.
11. $\frac{1}{8}$, $\frac{2}{3}$, and $\frac{3}{4}$.
12. $\frac{4}{5}$, $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{7}{10}$.

113. When there are mixed numbers, *the fractions and integral parts may be added separately; and then their sums may be added.*

13. What is the sum of $3\frac{1}{4}$, $6\frac{1}{2}$, and $2\frac{3}{8}$?

OPERATION.

$$\tfrac{1}{4}+\tfrac{1}{2}+\tfrac{3}{8}=\tfrac{2}{8}+\tfrac{4}{8}+\tfrac{3}{8}=\tfrac{9}{8}=1\tfrac{1}{8};$$
$$3+6+2=11;\ 11+1\tfrac{1}{8}=12\tfrac{1}{8}, \textit{Ans.}$$

What is the Rule? How may the addition be performed when *there are mixed numbers?*

14. What is the sum of $5\frac{2}{3}$, $6\frac{7}{8}$, and $4\frac{1}{2}$? Ans. $17\frac{1}{24}$.

15. What is the sum of $17\frac{1}{2}$, $2\frac{2}{2}$, and 3? Ans. $23\frac{1}{6}$.

16. Required the sum of $5\frac{4}{9}$, 6, and $7\frac{3}{7}$. Ans. $18\frac{5}{6}\frac{5}{3}$.

17. Bought a handkerchief for $\frac{5}{8}$ of a dollar, a vest for $2\frac{1}{2}$ dollars, a pair of gloves for $\frac{5}{16}$ of a dollar, and a hat for $4\frac{1}{4}$ dollars; how much did the whole cost? Ans. $\$7\frac{11}{16}$.

18. A merchant has sold from a piece of cloth, 12, $4\frac{2}{3}$, $8\frac{1}{7}$, and $2\frac{5}{7}$ yards; how many yards is this in all?

Ans. $27\frac{11}{21}$.

19. What is the sum of $\$9\frac{1}{5}$, $\$12\frac{1}{3}$, $\$10\frac{2}{3}$, and $\$7\frac{3}{4}$?

20. I have bought three lots of coal, weighing respectively, $1\frac{3}{5}$, $\frac{4}{15}$, and $9\frac{3}{20}$ tons; how much is there in all?

Ans. $11\frac{1}{60}$ tons.

21. A farmer sold sheep for $\$62\frac{7}{8}$, cattle for $\$102\frac{3}{5}$, and a horse for $\$125\frac{3}{4}$; how much did he receive for all?

SUBTRACTION.

114. One fraction may be subtracted from another, by means of a common denominator. Thus,

We can subtract *halves* from *halves*, *thirds* from *thirds*, *fourths* from *fourths*, etc.; but we can not subtract directly *thirds* from *halves*, *fourths* from *thirds*, etc., any more than we can subtract days from dollars, which are of different names or kinds.

1. Find the difference between $\frac{17}{18}$ and $\frac{14}{18}$.

OPERATION.

$\frac{17}{18} - \frac{14}{18} = \frac{17-14}{18} = \frac{3}{18} = \frac{1}{6}$, *Ans.*

The difference between 17 *eighteenths* and 14 *eighteenths*, is 3 *eighteenths*, or $\frac{3}{18}$, which, reduced, gives $\frac{1}{6}$. Therefore, the difference required is $\frac{1}{6}$.

How may an integer be expressed in a fractional form? (Art. 144.) How may one fraction be subtracted from another?

2. Find the difference between $\frac{7}{8}$ and $\frac{3}{4}$.

OPERATION.

$\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{7-6}{8} = \frac{1}{8}$, *Ans.*

Since, in order to subtract one number from another, the two numbers must be of the same name or kind, we reduce the given fractions to equivalent fractions having a common denominator, and have 7 *eighths* and 6 *eighths*, whose difference is 1 *eighth*, or $\frac{1}{8}$. Therefore, the difference required is $\frac{1}{8}$.

Rule.—*Reduce the fractions, if necessary, to equivalent fractions having a common denominator, and write the difference of the numerators over the common denominator.*

Examples.

What is the difference between

3. $\frac{5}{6}$ and $\frac{4}{5}$. Ans. $\frac{1}{30}$.
4. $\frac{8}{15}$ and $\frac{9}{20}$. Ans. $\frac{1}{12}$.
5. $\frac{3}{8}$ and $\frac{1}{2}$. Ans. $\frac{1}{8}$.
6. $\frac{7}{8}$ and $\frac{2}{8}$.
7. $\frac{1}{7}$ and $\frac{1}{11}$.
8. $\frac{15}{16}$ and $\frac{11}{12}$. Ans. $\frac{1}{48}$.
9. $\frac{11}{12}$ and $\frac{3}{4}$. Ans. $\frac{1}{6}$.
10. $\frac{15}{16}$ and $\frac{4}{7}$. Ans. $\frac{41}{112}$.
11. $\frac{16}{21}$ and $\frac{8}{15}$.
12. $\frac{3}{5}$ and $\frac{2}{7}$.

13. How much is $\frac{5}{9}$ greater than $\frac{4}{21}$? Ans. $\frac{23}{63}$.

14. Gave $\frac{5}{6}$ of a dollar for an arithmetic, and $\frac{2}{5}$ of a dollar for a slate; how much more did the arithmetic cost than the slate? Ans. $\frac{13}{30}$ of a dollar.

15. From 7 take $\frac{3}{4}$.

OPERATION.

$7 - \frac{3}{4} = \frac{28}{4} - \frac{3}{4} = \frac{25}{4} = 6\frac{1}{4}$, *Ans.*

Or, $7 - 1 = 6$; $1 = \frac{4}{4}$; $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$; $6 + \frac{1}{4} = 6\frac{1}{4}$, *Ans.*

Here, we change the whole number 7 to fourths, and subtract the $\frac{3}{4}$. We can, also, take 1 from the 7, leaving 6, reduce the 1 to fourths, subtract the $\frac{3}{4}$, and add the difference to the 6. The result will be the same by either process.

Recite the Rule. Explain the manner of working the 15th example.

16. Subtract $2\frac{6}{7}$ from 5. Ans. $2\frac{1}{7}$.

17. Subtract $5\frac{1}{2}$ from 9. Ans. $3\frac{1}{2}$.

18. Subtract $6\frac{1}{2}$ from $12\frac{5}{12}$.

OPERATION.

$$12\tfrac{5}{12} - 6\tfrac{1}{2} = 11\tfrac{17}{12} - 6\tfrac{6}{12} = 5\tfrac{11}{12}, \textit{Ans.}$$

Or,

$$12\tfrac{5}{12} - 6\tfrac{1}{2} = 11\tfrac{17}{12} - 6\tfrac{6}{12} = 5\tfrac{11}{12}, \textit{Ans.}$$

Here, by the second process, the fractional parts having been reduced to fractions having a common denominator, and as we can not take 6 twelfths from 5 twelfths, we take 1 from 12, leaving 11, reduce the 1 to twelfths, making 12 twelfths, add them to the $\frac{5}{12}$, giving $\frac{17}{12}$, and then, subtracting the $\frac{6}{12}$ from the $\frac{17}{12}$, and the 6 from the 11, we have $5\frac{11}{12}$, the difference required.

19. Subtract $2\frac{1}{2}$ from $5\frac{5}{8}$. Ans. $3\frac{1}{8}$.

20. Subtract $20\frac{1}{5}$ from $29\frac{3}{4}$.

21. If from a cask containing $25\frac{3}{8}$ gallons of wine, $7\frac{2}{3}$ gallons leak out, how much remains in it?

Ans. $17\frac{17}{24}$ gallons.

22. Bought a lot of coal for $31\frac{7}{8}$ dollars, and sold it for $37\frac{9}{10}$ dollars; how much was gained?

Ans. $6\frac{1}{40}$ dollars.

23. Paid $10\frac{3}{10}$ dollars for a barrel of flour, and sold it for $1\frac{7}{12}$ dollars less than cost, how much was obtained for it? Ans. $8\frac{43}{60}$ dollars.

24. When a ton of coal, bought for $5\frac{9}{10}$ dollars, is sold for $7\frac{1}{4}$ dollars, how much is gained? Ans. $1\frac{7}{20}$ dollars.

25. Paid $113\frac{7}{8}$ dollars for some goods, and sold them at $7\frac{3}{4}$ dollars under cost; for how much were they sold?

Ans. $106\frac{1}{8}$ dollars.

Explain the manner of working the 18th example.

MULTIPLICATION.

115. A fraction is multiplied by multiplying its numerator, or by dividing its denominator.

For, let it be required to multiply $\frac{1}{8}$ by 4. Then 4 times 1 eighth are 4 eighths, which, reduced, is $\frac{1}{2}$. Now, if instead of taking the *number* of parts 4 times, we make the *size* of the parts 4 times as large, by dividing the denominator 8 by 4, we have, also, $\frac{1}{2}$ as before.

Case I.

A Fraction by an Integer.

116. Multiply $\frac{4}{15}$ by 3.

OPERATION.

$\frac{4}{15} \times 3 = \frac{4 \times 3}{15} = \frac{12}{15} = \frac{4}{5}$, *Ans.*

Or, $\frac{4}{15} \times 3 = \frac{4}{15 \div 3} = \frac{4}{5}$, *Ans.*

3 times 4 *fifteenths*, or $\frac{4 \times 3}{15}$ is $\frac{12}{15}$, which reduced is $\frac{4}{5}$; or since dividing the denominator multiplies the fraction (Art. 115), we divide the denominator 15 by 3, and have $\frac{4}{15 \div 3}$, or $\frac{4}{5}$ as before.

Rule. — *Multiply the numerator by the integer; or, Divide the denominator by the integer, when it can be done without a remainder.*

Examples.

Multiply

2. $\frac{7}{11}$ by 4.	Ans. $2\frac{6}{11}$.	7. $\frac{15}{16}$ by 25.	Ans. $23\frac{7}{16}$.
3. $\frac{8}{16}$ by 7.	Ans. $3\frac{1}{2}$.	8. $\frac{3}{20}$ by 7.	Ans. $1\frac{1}{20}$.
4. $\frac{3}{4}$ by 16.	Ans. 12.	9. $\frac{18}{11}$ by 5.	Ans. $8\frac{2}{11}$.
5. $\frac{3}{5}$ by 18.		10. $\frac{19}{21}$ by 21.	
6. $\frac{7}{12}$ by 11.		11. $\frac{9}{13}$ by 26.	

How is a fraction multiplied? Explain the operation. What is *the Rule?*

12. At $\frac{7}{8}$ of a dollar a bushel, what cost 12 bushels of corn? Ans. $10\frac{1}{2}$.

117. When the multiplicand is a *mixed number*, we may *multiply the fractional part and integral part separately, and add the products;* or, *reduce the mixed number to an improper fraction, and then multiply.*

13. Required the product of $7\frac{3}{4}$ multiplied by 5.

OPERATION.

$\frac{3}{4} \times 5 = \frac{3 \times 5}{4} = \frac{15}{4} = 3\frac{3}{4}$

$7 \times 5 = 35$

$7\frac{3}{4} \times 5 = 38\frac{3}{4}$, *Ans.*

Or, $7\frac{3}{4} \times 5 = \frac{31 \times 5}{4} = \frac{155}{4} = 38\frac{3}{4}$, *Ans.*

14. Multiply $3\frac{2}{5}$ by 7. Ans. $23\frac{4}{5}$.

15. Multiply $8\frac{4}{7}$ by 6.

16. Multiply $10\frac{2}{3}$ by 9. Ans. 92.

17. Multiply $11\frac{3}{11}$ by 20.

18. If one man can build $15\frac{2}{3}$ rods of wall in a week, how many rods can 13 men build in the same time? Ans. $203\frac{2}{3}$.

Case II.

An Integer by a Fraction.

118. Multiplying a number by a fraction is taking the *part* of it denoted by the fraction.

For, multiplying any number, as 6, by $\frac{1}{2}$, is taking 1 *half* of 6, by $\frac{1}{3}$ is taking 1 *third* of 6, by $\frac{2}{3}$ is taking 2 *thirds* of 6, etc.

1. Let it be required to multiply 16 by $\frac{3}{4}$.

How may we multiply when the multiplicand is a mixed number? Explain the operation. What is multiplying a number by a fraction?

OPERATION.

$$16 \times \tfrac{3}{4} = \frac{16 \times 3}{4} = 12, \textit{ Ans.}$$

Since 1 *fourth* of 16 is 4, 3 *fourths* of 16 are 3 times 4, or 12. Or 16 taken 3 times is 16×3, and taken 1 *fourth* as many times is $\frac{16 \times 3}{4}$ or 12, as before.

Rule.—*Multiply the integer by the numerator, and divide the product by the denominator.*

Examples.

Multiply

2. 48 by $\frac{2}{3}$.	Ans. 32.	7. 64 by $\frac{1}{4}$.	Ans. $9\frac{1}{4}$.
3. 33 by $\frac{3}{11}$.	Ans. 9.	8. 25 by $\frac{15}{16}$.	Ans. $23\frac{7}{16}$.
4. 16 by $\frac{4}{5}$.	Ans. $12\frac{4}{5}$.	9. 416 by $\frac{3}{8}$.	Ans. 156.
5. 18 by $\frac{5}{6}$.		10. 23 by $\frac{5}{12}$.	
6. 51 by $\frac{2}{7}$.		11. 101 by $\frac{2}{9}$.	

12. What will cost 160 bushels of oats, at $\frac{5}{8}$ of a dollar a bushel? Ans. $100.

13. If an acre of land is worth 239 dollars, what are $\frac{2}{11}$ of it worth? Ans. $43\frac{5}{11}$.

14. What will cost 144 yards of cloth, at $\frac{9}{16}$ of a dollar per yard? Ans. $81.

119. When the multiplier is a *mixed number*, we may *multiply by the fractional and integral parts separately, and add the products; or, reduce the mixed number to an improper fraction, and then multiply.*

15. What is the product of 33 by $3\frac{2}{5}$?

Explain the operation. Recite the Rule. When the multiplier is a *mixed number*, how do you proceed?

OPERATION.

$$33 \times \frac{2}{5} = \frac{33 \times 2}{5} = \frac{66}{5} = 13\frac{1}{5}.$$

$$33 \times 3 \qquad\qquad 99$$

$$33 \times 3\frac{2}{5} \qquad\qquad 112\frac{1}{5}, \textit{Ans.}$$

Or, $33 \times 3\frac{2}{5} = 33 \times \frac{17}{5} = \frac{561}{5} = 112\frac{1}{5}$, *Ans.*

What is the product of

16. 12 by $9\frac{3}{5}$. Ans. $115\frac{1}{5}$.
17. 43 by $8\frac{5}{6}$.
18. 67 by $5\frac{2}{13}$. Ans. $345\frac{4}{13}$
19. 112 by $12\frac{3}{8}$.

20. If a car-load of coal is worth \$45, how much are $10\frac{4}{17}$ times that quantity worth? Ans. $460\frac{10}{17}$.

Case III.

A Fraction by a Fraction.

120. A **Fraction of a Fraction,** as $\frac{1}{3}$ of $\frac{3}{4}$, is called a *compound fraction;* and the multiplying of a fraction by a fraction is equivalent to finding the value of a compound fraction.

Thus, $\frac{3}{4} \times \frac{1}{2}$ is equivalent to taking $\frac{1}{2}$ of $\frac{3}{4}$.

1. Find the product of $\frac{4}{5}$ by $\frac{3}{8}$.

OPERATION.

$$\frac{4}{5} \times \frac{3}{8} = \frac{4 \times 3}{5 \times 8} = \frac{3}{10}, \textit{Ans.}$$

1 *eighth* of $\frac{4}{5}$ is $\frac{4}{5\times8}$, and 3 *eighths* of $\frac{4}{5}$ are $\frac{4\times3}{5\times8}$, or, by reducing, $\frac{3}{10}$.

What is a fraction of a fraction called? What is the multiplying of a fraction by a fraction equivalent to?

Rule.—*Multiply the numerators together for a new numerator, and the denominators for a new denominator.*

Examples.

Multiply

2. $\frac{4}{5}$ by $\frac{2}{3}$. Ans. $\frac{8}{15}$.
3. $\frac{8}{11}$ by $\frac{1}{8}$. Ans. $\frac{1}{11}$.
4. $\frac{5}{3}$ by $\frac{2}{5}$. Ans. $\frac{2}{3}$.
5. $1\frac{1}{2}$ by $\frac{6}{7}$.
6. $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{2}$. Ans. $\frac{1}{12}$.
7. $\frac{7}{8}$ by $\frac{1}{5}$ of $\frac{3}{7}$. Ans. $\frac{3}{40}$.
8. $1\frac{1}{4}$ by $\frac{7}{4} \times \frac{2}{11}$. Ans. $\frac{1}{4}$.
9. $\frac{9}{15} \times \frac{4}{3}$ by $\frac{2}{5}$.

10. What part of a ship is $\frac{1}{4}$ of $\frac{5}{8}$ of it? Ans. $\frac{5}{32}$.

11. What is the product of $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{4}{5}$?

12. At $\frac{8}{25}$ of a dollar a yard, what will $\frac{5}{16}$ of a yard of cloth cost? Ans. $\frac{1}{10}$ of a dollar.

121. When there are *integral* or *mixed numbers* among the factors, *reduce them to improper fractions before multiplying.*

13. What is the product of $\frac{5}{7}$ of $4\frac{1}{10}$ by 7?

OPERATION.

$$\frac{5}{7} \times 4\frac{1}{10} \times 7 = \frac{5 \times 41 \times 7}{7 \times 10 \times 1} = \frac{41}{2} = 20\frac{1}{2}, \textit{Ans.}$$

14. What is the product of $7\frac{1}{2}$ by $9\frac{1}{4}$? Ans. $69\frac{3}{8}$.

15. What will $5\frac{1}{4}$ pounds of sugar cost, at $\frac{1}{6}$ of a dollar a pound? Ans. $\frac{7}{8}$ of a dollar.

16. What will $50\frac{5}{16}$ pounds of beef cost, at $\frac{4}{25}$ of a dollar a pound? Ans. $\$8\frac{1}{20}$.

17. What is the value of $\frac{3}{4}$ of $\frac{7}{8}$ of 9? Ans. $5\frac{23}{32}$.

18. When cloth is $\frac{7}{8}$ of a dollar a yard, what will $\frac{4}{7}$ of 12 yards cost? Ans. $6.

Recite the Rule. When there are integral or mixed numbers *among the factors*, how do you proceed?

19. What is the value of $\frac{9}{10}$ of $\frac{5}{27}$ of $6\frac{3}{5}$?

20. At $\frac{9}{16}$ of a dollar a bushel, what cost $\frac{4}{18}$ of 32 bushels of apples? Ans. $4.

DIVISION.

122. A fraction is divided by dividing its numerator, or by multiplying its denominator.

For, let it be required to divide $\frac{4}{8}$ by 4. Then, 1 fourth of 4 eighths, or $\frac{4 \div 4}{8}$, is $\frac{1}{8}$; or, if instead of finding 1 fourth of the *number* of parts, we make the *size* of the parts 1 fourth as large, by multiplying the denominator 8 by 4, we have $\frac{4}{8 \times 4}$, or, by reducing, $\frac{1}{8}$, as before.

Case I.

A Fraction by an Integer.

123. 1. Let it be required to divide $\frac{18}{23}$ by 6.

OPERATION.

$$\frac{18}{23} \div 6 = \frac{18 \div 6}{23} = \frac{3}{22}, \text{ Ans.}$$

Since $\frac{18}{23}$ divided by 1 is $\frac{18}{23}$, $\frac{18}{23}$ divided by 6 is $\frac{1}{6}$ of $\frac{18}{23}$, which is $\frac{18 \div 6}{23}$, or $\frac{3}{23}$. Therefore, the quotient of $\frac{18}{23}$ divided by 6 is $\frac{3}{23}$.

2. What is the quotient of $\frac{1}{4}$ divided by 3?

OPERATION.

$$\frac{1}{4} \div 3 = \frac{1}{4 \times 3} = \frac{1}{12}, \text{ Ans.}$$

Since $\frac{1}{4}$ divided by 1 is $\frac{1}{4}$, $\frac{1}{4}$ divided by 3 is $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{4 \times 3}$, or $\frac{1}{12}$. Therefore, the quotient of $\frac{1}{4}$ divided by 3 is $\frac{1}{12}$.

How is a fraction divided? Explain the operations.

Rule. — *Divide the numerator by the integer, when it can be done without a remainder; or,*
Multiply the denominator by the integer.

Examples.

Divide

3. $\frac{7}{8}$ by 4. Ans. $\frac{7}{32}$.
4. $\frac{21}{25}$ by 7. Ans. $\frac{3}{25}$.
5. $\frac{48}{37}$ by 12.
6. $\frac{7}{40}$ by 21.
7. $\frac{18}{20}$ by 27. Ans. $\frac{1}{30}$.
8. $\frac{24}{47}$ by 16. Ans. $\frac{3}{94}$.
9. $\frac{36}{65}$ by 18.
10. $\frac{45}{63}$ by 25.

11. How much is $\frac{8}{15}$ of $\frac{3}{4}$ divided by 10?

OPERATION.

$$\frac{\overset{2}{\cancel{8}} \times \cancel{3}}{\underset{5}{\cancel{15}} \times \cancel{4} \times \underset{5}{\cancel{10}}} = \frac{1}{25}, \text{ Ans.}$$

12. If 6 yards of cloth cost $\frac{15}{16}$ of a dollar, what will 1 yard cost? Ans. $\frac{5}{32}$ of a dollar.

13. When 11 pounds of coffee can be bought for $\frac{3}{16}$ of a dollar, how much is it a pound?

124. When the dividend is a *mixed number, reduce it to an improper fraction before dividing.*

14. Find the quotient of $5\frac{1}{7}$ divided by 12.

OPERATION.

$$5\tfrac{1}{7} \div 12 = \frac{36}{7} \div 12 = \frac{\overset{3}{\cancel{36}}}{7 \times \underset{1}{\cancel{12}}} = \frac{3}{7}, \text{ Ans.}$$

Recite the Rule. When the dividend is a mixed number, how do you proceed?

15. $14\frac{3}{4}$ by 8. Ans. $1\frac{7}{32}$.
16. $21\frac{3}{7}$ by 25.
17. $4\frac{11}{13}$ by 9. Ans. $\frac{1}{13}$.
18. $30\frac{1}{4}$ by 22.

19. When 4 bushels of peaches cost $\$4\frac{1}{2}$, what is the cost of 1 bushel? Ans. $\$1\frac{1}{8}$.

20. If $\$28\frac{4}{5}$ be divided equally between 12 men, how much will each receive? Ans. $\$2\frac{2}{5}$.

21. How much is $\frac{5}{8}$ of 11 dollars divided by 22?
Ans. $\frac{5}{16}$ of a dollar.

22. At \$15 a ton, how many tons of hay can be bought for $\$116\frac{1}{4}$? Ans. $7\frac{3}{4}$ tons.

Case II.

An Integer by a Fraction.

125. 1. Let it be required to find the quotient of 12 divided by $\frac{1}{4}$.

OPERATION.

$$12 \div \tfrac{1}{4} = \frac{12 \times 4}{1} = 48, \textit{ Ans.}$$

Since 12 divided by 1 is 12 divided by $\frac{1}{4}$ is 4 times 12, or 48. Therefore, the quotient of 12 divided by $\frac{1}{4}$ is 48.

2. Find the quotient of 16 divided by $\frac{2}{5}$.

OPERATION.

$$16 \div \tfrac{2}{5} = \frac{16 \times 5}{2} = 40, \textit{ Ans.}$$

Since 16 divided by 1 is 16, 16 divided by $\frac{1}{5}$ is 5 times 16, and 16 divided by 2 fifths is $\frac{1}{2}$ of 5 times 16, which is $\frac{5}{2}$ times 16, or 40. Therefore, the quotient of 16 divided by $\frac{2}{5}$ is 40.

Rule.—*Divide by the numerator of the divisor, and multiply by the denominator; or,*

Explain the operation of Example 1. Of Example 2. Recite the Rule.

Invert the divisor, and proceed as in multiplication.

Examples.

Divide

3. 8 by $\frac{4}{5}$. Ans. 10.
4. 14 by $\frac{2}{2}\frac{4}{5}$. Ans. $14\frac{7}{12}$.
5. 3 by $\frac{8}{10}$. Ans. $3\frac{1}{3}$.
6. 15 by $\frac{10}{11}$.
7. 75 by $\frac{3}{4}$. Ans. 100.
8. 5 by $\frac{7}{10}$. Ans. $7\frac{1}{7}$.
9. 27 by $\frac{5}{8}$. Ans. $48\frac{3}{5}$.
10. 34 by $\frac{17}{20}$.

11. At $\frac{3}{16}$ of a dollar a pound, how much coffee can be bought for $12? Ans. 64 pounds.

12. If it takes $\frac{2}{3}$ of a yard of cloth to make a vest, how many vests can be made from 6 yards? Ans. 9 vests.

126. When the divisor is a *mixed number*, *reduce it to an improper fraction*, *before dividing*.

13. Divide 12 by $1\frac{1}{2}$.

OPERATION.

$$12 \div 1\tfrac{1}{2} = 12 \times \tfrac{2}{3} = \frac{\overset{4}{\cancel{12}} \times 2}{\cancel{3}} = 8, \textit{ Ans.}$$

14. 75 by $3\frac{2}{3}$. Ans. $20\frac{5}{11}$.
15. 18 by $1\frac{1}{4}$.
16. 92 by $5\frac{1}{3}$. Ans. 18.
17. 100 by $4\frac{7}{8}$.

18. At $6\frac{1}{4}$ cents a pound, how many pounds of rice can be bought for $1.50? Ans. 24 pounds.

19. How many bushels of wheat, at $\$2\frac{1}{8}$ a bushel, can be bought for $39? Ans. 18 bushels.

When the divisor is a mixed number, how do you proceed? Explain the operation.

Case III.

A Fraction by a Fraction.

127. 1. Let it be required to divide $\frac{4}{5}$ by $\frac{2}{3}$.

OPERATION.

$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}, \textit{ Ans.}$$

Since $\frac{4}{5}$ divided by 1 is $\frac{4}{5}$, $\frac{4}{5}$ divided by $\frac{1}{3}$ is 3 times $\frac{4}{5}$, and $\frac{4}{5}$ divided by 2 thirds is $\frac{1}{2}$ of 3 times $\frac{4}{5}$, which is $\frac{3}{2}$ times $\frac{4}{5}$, or $\frac{12}{10}$; or, by reducing, $1\frac{1}{5}$. Therefore, the quotient of $\frac{4}{5}$ divided by $\frac{2}{3}$ is $1\frac{1}{5}$.

Rule.—*Divide by the numerator of the divisor, and multiply by its denominator; or,*

Invert the divisor, and proceed as in multiplication.

If there are mixed numbers, reduce them to improper fractions.

Divide

2. $\frac{7}{8}$ by $\frac{1}{2}$.	Ans. $1\frac{3}{4}$.	7. $\frac{9}{30}$ by $4\frac{1}{2}$.	Ans. $\frac{1}{15}$.
3. $\frac{2}{7}$ by $\frac{4}{11}$.	Ans. $\frac{11}{14}$.	8. $3\frac{1}{6}$ by $9\frac{1}{2}$.	Ans. $\frac{1}{3}$.
4. $\frac{5}{6}$ by $\frac{2}{9}$.	Ans. $3\frac{3}{4}$.	9. $\frac{7}{16}$ by $\frac{7}{8}$.	Ans. $\frac{1}{2}$.
5. $\frac{8}{9}$ by $\frac{4}{7}$.		10. $9\frac{1}{2}$ by $\frac{7}{8}$.	
6. $\frac{9}{13}$ by $\frac{3}{4}$.		11. $6\frac{1}{2}$ by $3\frac{1}{3}$.	

12. How many pounds of tea, at $\frac{1}{4}$ of a dollar a pound, can be bought for $\$7\frac{1}{2}$? Ans. 30 pounds.

13. When for $\frac{7}{8}$ of a dollar $3\frac{1}{2}$ bushels of apples can be bought, what is the cost of 1 bushel?

14. At $\frac{7}{8}$ of a dollar for $\frac{2}{5}$ of a yard of cloth, what is the price per yard? Ans. $1\frac{7}{8}$ dollars.

REVIEW. — What is an Integer? (84.) What is a Prime Number? (85.) What is a Composite Number? (86.)

15. At $\frac{5}{8}$ of a dollar a gallon, how many gallons of kerosene can be bought for $\$5\frac{1}{2}$?

16. How many bottles, of $1\frac{1}{2}$ pints each, can be filled with $27\frac{3}{4}$ gallons of wine? Ans. 148 bottles.

REVIEW EXERCISES.

1. Reduce $\frac{144}{240}$ to its lowest terms. Ans. $\frac{3}{5}$.
2. Reduce $\frac{23}{92}$ to its lowest terms.
3. Change \$31 to sixths of a dollar. Ans. $\$\frac{186}{6}$.
4. Reduce $19\frac{3}{7}$ to an improper fraction. Ans. $\frac{136}{7}$.
5. Change 22 to a fractional form. Ans. $\frac{22}{1}$.
6. Change 22 to a fraction whose denominator is 3.
7. Reduce $\frac{1}{3}$ and $\frac{1}{5}$ to fractions having a common denominator.
8. Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$. Ans. $1\frac{29}{36}$.
9. Find the sum of $\frac{3}{5}$, $2\frac{3}{10}$, and $5\frac{1}{4}$. Ans. $8\frac{23}{20}$.
10. How much larger is $\frac{8}{15}$ than $\frac{9}{20}$? Ans. $\frac{1}{12}$.
11. Find the difference between $\frac{10}{11}$ and $\frac{6}{7}$.
12. From $6\frac{1}{14}$ subtract $2\frac{11}{21}$. Ans. $4\frac{11}{42}$.
13. Change $\frac{131}{5}$ to a mixed number. Ans. $26\frac{1}{5}$.
14. Find the value of $\frac{13}{14} \times 6$. Ans. $5\frac{4}{7}$.
15. Multiply 51 by $\frac{5}{17}$. Ans. 15.
16. How much is $\frac{8}{9}$ of $\frac{1}{2}$ of $\frac{6}{7}$? Ans. $\frac{8}{21}$.
17. Find the value of $\frac{3}{11} \times \frac{7}{4} \times \frac{8}{3}$.
18. Find the value of $\frac{2}{9} + \frac{5}{4} - 1\frac{1}{12}$. Ans. $\frac{7}{18}$.
19. Divide $\frac{17}{20}$ by 9. Ans. $\frac{17}{180}$.
20. Divide 26 by $\frac{13}{15}$. Ans. 30.
21. How many times is $\frac{7}{8}$ contained in 4?
22. What is the value of $\frac{12}{17} \div \frac{4}{5}$? Ans. $\frac{15}{17}$.

REVIEW. — What is Analysis? (94.) What is a Fraction? (96.) A Common Fraction? (142.) The Terms of a Fraction? (99.)

23. A merchant owning $\frac{3}{8}$ of a ship, bought $\frac{5}{16}$ more; how much did he then own of it? Ans. $\frac{11}{16}$.

24. A farmer divided some hay into 20 equal parts, and gave 16 parts to his cows, and the rest to his sheep; what fraction of the whole did the sheep get? Ans. $\frac{1}{5}$.

25. What number is that to which if you add $3\frac{5}{9}$, the sum will be $16\frac{5}{12}$? Ans. $12\frac{31}{36}$.

26. If $\frac{3}{8}$ of a mill is worth \$3740, what is the value of the whole? Ans. $9973\frac{1}{3}$.

27. When 13 tons of coal are sold for $\$71\frac{1}{2}$, what is the price per ton? Ans. $5\frac{1}{2}$.

28. Paid $\$10\frac{1}{2}$ for tea, at the rate of $\frac{7}{10}$ of a dollar a pound; how many pounds did I buy?

29. If the divisor is $\frac{5}{13}$, and the quotient $2\frac{1}{4}$, what is the dividend? Ans. $4\frac{5}{52}$.

30. How many bushels of corn, at $\frac{9}{10}$ of a dollar a bushel, must be given for $1\frac{3}{5}$ of a ton of coal, at $\$4\frac{1}{2}$ a ton? Ans. 8.

31. When broadcloth is $4\frac{3}{8}$ dollars a yard, how many yards can be bought for 3 cords of wood, at 7 dollars a cord? Ans. $4\frac{4}{5}$ yards.

QUESTIONS FOR ANALYSIS.

1. If 1 barrel of flour cost \$9, how much will $\frac{3}{4}$ of a barrel cost?

SOLUTION. — *If* 1 *barrel cost* \$9, $\frac{1}{4}$ *of a barrel will cost* $\frac{1}{4}$ *of* \$9, *or* \$2.25; *and* $\frac{3}{4}$ *will cost* 3 *times* \$2.25, *or* \$6.75. *Therefore, if* 1 *barrel of flour cost* \$9, $\frac{3}{4}$ *of a barrel will cost* \$6.75.

2. If 1 ton of coal be worth \$5.25, what is $\frac{4}{5}$ of a ton worth? Ans. \$4.20.

3. When \$8.80 is paid for doing a certain amount of work, how much must be paid for doing $\frac{7}{11}$ as much?

4. If an acre of land produce $36\frac{9}{50}$ hundred weight of hay, how much will $\frac{8}{9}$ of an acre produce?

5. If in a lot there are $15\frac{37}{40}$ acres, what is the extent of $\frac{8}{9}$ of it? Ans. $11\frac{3}{8}$ acres.

6. If $\frac{3}{4}$ of a barrel of flour cost \$6.75, what will 1 barrel cost?

SOLUTION. — *If* $\frac{3}{4}$ *of a barrel cost* \$6.75, $\frac{1}{4}$ *of a barrel will cost* $\frac{1}{3}$ *of* \$6.75, *or* \$2.25, *and* $\frac{4}{4}$, *or* 1 *barrel, will cost* 4 *times* \$2.25, *or* \$9. *Therefore, if* $\frac{3}{4}$ *of a barrel of flour cost* \$6.75, 1 *barrel will cost* \$9.

7. If $\frac{7}{8}$ of a yard of cloth cost \$3.78, what will a whole yard cost? Ans. \$4.32.

8. Paid \$156 for a horse, which was $1\frac{3}{13}$ as much as I paid for a wagon; how much did I pay for the wagon? Ans. \$180.

9. A farmer sold $31\frac{1}{8}$ bushels of grain, which was $\frac{7}{8}$ of all he had; how much had he? Ans. $36\frac{1}{2}$ bushels.

10. If $11\frac{1}{32}$ acres is $\frac{4}{7}$ of a certain field, what is its entire extent? Ans. $15\frac{47}{80}$ acres.

11. If 4 tons of hay cost \$52, what will $3\frac{2}{13}$ tons cost?

SOLUTION. — *If* 4 *tons cost* \$52, 1 *ton will cost* $\frac{1}{4}$ *of* \$52, *or* \$13. $3\frac{2}{13}$ *tons are* $\frac{41}{13}$ *tons; and if* 1 *ton cost* \$13, $\frac{1}{13}$ *will cost* $\frac{1}{13}$ *of* \$13, *or* \$1, *and* $\frac{41}{13}$ *will cost* 41 *times* \$1, *or* \$41. *Therefore, if* 4 *tons of hay cost* \$52, $3\frac{2}{13}$ *tons will cost* \$41.

12. When \$60 are paid for 12 yards of broadcloth, how much must be paid for $9\frac{7}{8}$ yards? Ans. \$49.37$\frac{1}{2}$.

13. If $3\frac{2}{13}$ *tons* of hay cost \$41, what will 4 tons cost?

14. If 9 bushels of wheat are worth \$19.35, what are $10\frac{2}{5}$ bushels worth? Ans. \$22.79.

15. If 17 bushels of corn cost \$16.32, what will $15\frac{3}{4}$ bushels cost? Ans. \$15.12.

16. If $\frac{2}{3}$ of an acre of land is worth \$60, what is $\frac{5}{6}$ of an acre worth?

SOLUTION. — *If* $\frac{2}{3}$ *of an acre is worth* \$60, $\frac{1}{3}$ *is worth* $\frac{1}{2}$ *of* \$60, *or* \$30, *and* $\frac{3}{3}$, *or* 1 *acre, is worth* 3 *times* \$30, *or* \$90; *and if* 1 *acre is worth* \$90, $\frac{1}{6}$ *of an acre is worth* $\frac{1}{6}$ *of* \$90, *or* \$15, *and* $\frac{5}{6}$ *is worth* 5 *times* \$15, *or* \$75. *Therefore, if* $\frac{2}{3}$ *of an acre is worth* \$60, $\frac{5}{6}$ *of an acre is worth* \$75.

17. If $\frac{3}{8}$ of a cord of wood cost \$2.85, what will $\frac{3}{4}$ of a cord cost? Ans. \$5.70.

18. When $\frac{1}{4}$ of a pound of tea can be bought for \$0.54, what will $4\frac{3}{5}$ pounds cost? Ans. \$3.12.

19. If $4\frac{1}{2}$ pounds of butter are worth \$1.44, what are $11\frac{7}{16}$ pounds worth? Ans. 3.66.

DECIMAL FRACTIONS.

128. If a unit be divided into ten equal parts, each of these parts will be 1 *tenth*.

If each tenth be divided into ten equal parts, each part will be $\frac{1}{10}$ of $\frac{1}{10}$, or 1 *hundredth*.

If each hundredth be divided into ten equal parts, each part will be $\frac{1}{10}$ of $\frac{1}{100}$, or 1 *thousandth*.

In like manner, we may, by dividing by ten, continue to obtain fractions, each of whose values is *one tenth* of the fraction preceding it.

129. A **Decimal Fraction**, or a **Decimal**, is a fraction whose unit is divided into *tenths*, *hundredths*, *thousandths*, etc.

NOTATION AND NUMERATION.

130. A decimal fraction, by means of the decimal point (.), is written without the denominator, and the value of the decimal figures determined by the place of each with reference to the decimal point.

The first order to the right of the decimal point is that of *tenths*. Thus,

$\frac{1}{10}$ may be written .1, and read 1 tenth;
$\frac{2}{10}$ " " .2, " 2 tenths;
$\frac{3}{10}$ " " .3, " 3 tenths;

and so on.

The second order to the right of the decimal point is that of *hundredths*. Thus,

$\frac{1}{100}$ may be written .01, and read 1 hundredth;
$\frac{2}{100}$ " " .02, " 2 hundredths;
$\frac{3}{100}$ " " .03, " 3 hundredths;

and so on.

The third order to the right of the decimal point is that of *thousandths*. Thus,

$\frac{1}{1000}$ may be written .001, and read 1 thousandth;
$\frac{2}{1000}$ " " .002, " 2 thousandths;
$\frac{3}{1000}$ " " .003, " 3 thousandths;

and so on.

What is a Decimal Fraction? How is it written? What is the *first order to the right* of the point? The second? The third?

The fourth order to the right is *ten-thousandths;* the fifth, *hundred-thousandths*, and so on.

131. Since decimals of the first order are *tenths*, of the second order *hundredths*, of the third order *thousandths*, etc., it follows that

The denominator of a decimal fraction is 1 *with as many ciphers annexed as the numerator has decimal places.*

132. Since each lower order is one tenth of the next higher, it follows, in decimals, as in simple whole numbers, that

Ten of any lower order are always equal to one of the next higher.

Hence integers and decimals may be expressed together as a mixed number, by placing the decimal point between them. Thus,

17 and 25 hundredths may be written 17.25.

An integer may be regarded as a decimal by placing the decimal point on the right of the order of *units;* and the expression may be read, according to the decimal orders annexed, as a number of *tenths*, *hundredths*, etc. Thus,

17 may be written 17.0, 17.00, etc., and read 170 *tenths*, 1700 *hundredths*, etc.

133. The orders of decimals are named from the decimal point to the right; and a decimal may be separated into periods of three figures each, annexing ciphers, if required, to complete the last period. Hence the

What is the fourth order to the right of the point? How are the orders named?

Table.

Integers.							Decimals.					
Thousands.			Units.				Thousandths.			Millionths.		
of			of				of			of		
Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point.	Hundreds	Tens	Units	Hundreds	Tens	Units
3	4	6 ,	7	8	2	.	5	0	9 ,	1	3	0

where the expression may be read, three hundred forty-six *thousands*, seven hundred eighty-two *units*, and five hundred nine *thousandths*, one hundred thirty *millionths*.

The decimal part without the annexed cipher may be read fifty thousand nine hundred thirteen *hundred-thousandths*.

134. In United States Money, the dollar is the unit, and cents and mills are decimal parts of a dollar.

Thus, $5.25 is the same as 5 dollars and 25 hundredths of a dollar.

Case I.

Reading Decimals expressed by Figures.

135. 1. Write and read the decimal .035.

Solution. — The figures are 0 tenth, 3 hundredths, and 5 thousandths; or, since 3 hundredths equal 30 thousandths, and 30 thousandths plus 5 thousandths, equal 35 thousandths, we have, as the value expressed, 35 thousandths. Therefore, .035 is read *thirty-five thousandths*.

Give the mixed number expressed in the Table. How may cents and mills be regarded?

Rule.—*Read the decimal as an integer, giving it the name of the right-hand order.*

Examples.

Write and read the following decimals:—

2. .15.	6. .5.	10. .506.
3. .05.	7. .75.	11. .7153.
4. .005.	8. .603.	12. .0174.
5. .125.	9. .1325.	13. .006501.

136. When the decimal is part of a mixed number, we may *read the integral part as expressing units.* Thus,

16.45 may be read, *sixteen units and forty-five hundredths.*

109.005 may be read, *one hundred nine units and five thousandths.*

Write and read the following mixed numbers:—

14. 4.026.	17. 35.1675.	20. 5067.5.
15. 21.006.	18. 91.289.	21. 183.57.
16. 15.15.	19. 102.05.	22. 4.30068.

Case II.

Writing Decimals in Figures.

137. 1. Express in figures thirteen thousandths.

SOLUTION.—Since thirteen thousandths equal 1 hundredth and 3 thousandths, we write 3 in the order of thousandths, 1 in the order of hundredths, and, as there are no tenths, 0 in the order of tenths; and, prefixing the decimal point, we have .013.

Recite the Rule.

Rule. — *Write the figures at the right of the decimal point, each in its proper order, noting vacant orders, if any, by ciphers.*

Examples.

Write in the decimal form: —

2. $\frac{7}{10}$. Ans. .7.
3. $\frac{8}{100}$. Ans. .03.
4. $\frac{5}{1000}$.
5. $\frac{65}{1000}$.
6. $\frac{106}{1000}$. Ans. .106.
7. $\frac{7}{1000}$. Ans. .007.
8. $\frac{14}{10000}$.
9. $\frac{1068}{10000}$.
10. $7\frac{16}{100}$. Ans. 7.16.
11. $19\frac{9}{1000}$. Ans. 19.009.
12. $103\frac{21}{100}$.
13. $162\frac{121}{10000}$.

14. Three hundred six thousandths. Ans. .306.
15. Twenty-three ten-thousandths. Ans. .0023.
16. Thirty-six units, and five tenths. Ans. 36.5.
17. One hundred one units, and sixty-five hundredths. Ans. 101.65.
18. Sixteen units, and sixteen thousandths. Ans. 16.016.
19. Three hundred twenty-five ten-thousandths.
20. Five units, and three hundred thousandths.
21. Nineteen units, and six hundred thirty-one ten-thousandths.

REDUCTION.

138. *Annexing a cipher to a decimal does not alter the value of the decimal.*

For, the order of the significant figures of the decimal is not changed. Thus, .3, or .30, is the same as $\frac{3}{10}$.

139. Hence, to change decimals having different denominators, to equivalent decimals having a common denominator,

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Recite the Rule. What effect has the annexing of a cipher to a decimal? How do you change decimals having different denominators to equivalent decimals having a common denominator?

Make each decimal have the same number of decimal orders, by annexing ciphers.

EXERCISES.

Change to equivalent fractions having a common denominator,

1. .7 and .05.
   Ans. .70 and .05.
2. .06 and .103.
3. .5216 and .16.
4. .0031 and .316.
   Ans. .0031 and .3160.
5. .161 and .01.
6. .8 and .09163.

Case I.

A Decimal to a Common Fraction.

140. 1. Change .35 to a common fraction.

OPERATION.

$.35 = \frac{35}{100} = \frac{7}{20}$, *Ans.*

Removing the decimal point and writing the denominator, we have $\frac{35}{100}$, which, reduced, is $\frac{7}{20}$. Therefore, .35 is equal to $\frac{7}{20}$.

Rule. — *Write the denominator to the decimal, omit the decimal point, and reduce the common fraction to its lowest terms.*

Examples.

Reduce to common fractions: —

2. .75. Ans. $\frac{3}{4}$.
3. .375. Ans. $\frac{3}{8}$.
4. .06.
5. .008. Ans. $\frac{1}{125}$.
6. .0075. Ans. $\frac{3}{400}$.
7. .0625.

Case II.

A Common Fraction to a Decimal.

141. 1. Change $\frac{3}{4}$ to a decimal fraction.

Recite the Rule for reducing a decimal to a common fraction.

OPERATION.

4)3.00

75, *Ans.*

¾ equals 3 ÷ 4. Since we can not divide 3 by 4, we reduce 3 to tenths, and have 30 tenths; ¼ of 30 tenths is 7 tenths, with a remainder of 2 tenths; we write the 7 tenths. The 2 tenths we reduce to hundredths, and have 20 hundredths; ¼ of 20 hundredths is 5 hundredths, which we write. Therefore, ¾ equals .75.

Rule.—*Annex ciphers to the numerator, divide by the denominator, and point off as many orders for decimals as there were ciphers annexed.*

Examples.

Reduce to decimals:—

2. $\frac{5}{8}$.	Ans. .625	5. $\frac{17}{20}$.	
3. $\frac{1}{25}$.	Ans. .04.	6. $\frac{3}{125}$	Ans. .024.
4. $\frac{3}{8}$.		7. $\frac{3}{5}$.	Ans. 6.

142. When a common fraction can not be exactly expressed in decimals, the division may be carried to a sufficient degree of exactness, and the sign + annexed to the result to indicate its incompleteness.

8. Reduce $\frac{4}{9}$ to a decimal of three places.

Ans. .444+.

9. Reduce $\frac{6}{7}$ to a decimal of four places.

Ans. .8571+.

10. Reduce $\frac{5}{12}$ to a decimal of four places.

Ans. .4166+.

ADDITION.

143. Since ten of any order of decimals make one of the order next higher, decimals may be added in the same manner as simple integers. Hence,

Recite the Rule for reducing a common fraction to a decimal. *Why may decimals* be added the same as simple integers?

Rule.— *Write the numbers so that figures of the same order shall stand in the same column, add as in integers, and place the decimal point in the amount under those in the numbers added.*

Examples.

	(1.)	(2.)	(3.)
Add	2.35	41.144	.0632
	42.08	13.755	.456
	9.52	2.50	.81
	18.005	25.3	3.08
Ans.	71.955	82.699	4.4092

4. Add 6.051, .095, 31.6, and 1.0075.

5 Add 13.015, .9031, 12.186, and .09.

6. Add 120.16, .003, 1401.7, 1.5, and 19.8.

7. Add 7 units and 8 thousandths, 28 units and 16 hundredths, 56 units and 7 tenths. Ans. 91.868.

8. A merchant bought four lots of goods; for the first he paid $69.125; for the second, $193.3; for the third, $1008.56; and for the fourth, $752.375; how much did he pay for the whole? Ans. $2023.36.

SUBTRACTION.

144. Since ten units of any order of decimals make one of the order next higher, decimals may be subtracted in the same manner as simple integers. Hence,

Rule.— *Write the less number under the greater, so that figures of the same order shall stand in the same column; subtract as in integers, and place the*

Recite the Rule. Why may decimals be subtracted as in whole numbers? Recite the Rule.

decimal point in the difference under those in the numbers above.

Examples.

	(1.)	(2.)	(3.)
From	17.083	97.031	106.57
Take	9.36	18.346	41.139
Ans.	7.723		65.431

Here, in Example 3, as there are no thousandths in the minuend to subtract from, we consider that order in the minuend as filled by 0, since annexing a cipher to a decimal does not alter its value.

4. Subtract 5.91 from 13.675. Ans. 7.765.
5. Subtract 18.16 from 19.5. Ans. 1.34.
6. From 37 dollars take 31.375 dollars.
7. Bought 160 acres of land, and sold from it 141.125 acres; how much then had I left?
8. From 15 take 15 hundredths. Ans. 14.85.
9. A farmer owning 78 hundredths of a farm, sold 725 thousandths of it; how much of it had he left?

Ans. .055.

10. From 10 take 1 ten-thousandth. Ans. 9.9999.
11. Subtract 139.216 from 400.95. Ans. 261.734.
12. What is the difference between 64.075 and .195326?
13. From 107 take .0007. Ans. 106.9993.
14. From forty-three units and seventy-five thousandths, take thirty-five units and sixty-seven hundred-thousandths. Ans. 8.07433.

Is the value of a decimal altered by annexing a cipher?

MULTIPLICATION.

145. *Each removal of the decimal point one order toward the right multiplies by* 10.

For, each figure is made by the removal to denote units of an order next higher; hence the value expressed is made tenfold (Art. 33).

Thus, $.08 \times 10 = .8$; $.8 \times 10 = 8$; $.306 \times 100 = 30.6$.

146. *Each removal of the decimal point one order toward the left divides by* 10.

For, each figure is made by the removal to denote units of an order next lower; hence, the value expressed is made one tenth as much as it was.

Thus, $6.5 \div 10 = .65$; $.65 \div 10 = .065$; $73.4 \div 100 = .734$

One or both of the Factors Decimals.

147. 1. Multiply 3.16 by .4.

OPERATION.

$$\begin{array}{r} 3.16 \\ .4 \\ \hline \text{Ans. } 1.264 \end{array}$$

3.16 multiplied by 4 gives 12.64; but as the multiplier is .4, the product must be only a tenth as large, and 12.64 divided by 10, by removing the decimal point one order to the left, gives 1.264. Therefore, the product of 3.16 by .4 is 1.264.

Rule.—*Multiply as in integers, and point off as many figures for decimals in the product as there are in both multiplicand and multiplier, supplying the deficiency, if any, by prefixing ciphers; or,*

What effect has the removal of a decimal point one order toward the right? One order toward the left? Explain the operation. Recite the Rule.

If the multiplier is a decimal, multiply by its numerator, and divide by its denominator.

When the multiplier is 10, 100, 1000, etc., remove the point to the right as many orders as there are ciphers on the right of the multiplier.

Examples.

	(2.)	(3.)	(4.)
Multiply	.123	3162	.7869
By	.6	.08	100
Ans.	.0738	252.96	78.69

Multiply

5. 3.21 by 2.31. Ans. 7.4151.	9. .001 by .01. Ans. .00001.	13. .036 by .08. Ans. .00288.
6 1000 by .7. Ans. 700.	10. 78 by .42. Ans. 32.76.	14. 75 by .8. Ans. 60.
7. .106 by .35.	11. 9.16 by 10.	15. .009 by .009.
8. 3.18 by 41.7.	12. 65 by .65.	16. .063 by 1000.

17. What cost .65 of an acre of land, at $84 an acre? Ans. $54.60.

18. Multiply 12 hundredths by 8 thousandths.

19. What cost 18.75 barrels of flour, at $6.75 a barrel? Ans. $126.5625.

20. Find the value of .1 × .1 × .1. Ans. .001.

21. Find the value of .0002 × 3.01. Ans. .000602.

22. What cost 18.25 yards of cambric, at $0.125 a yard? Ans. $2.28125.

23. When $14.56 is the cost of a load of coal, what is the cost of 2.25 loads? Ans. $32.76.

How do you multiply by 10, 100, 1000, etc.?

24. Multiply thirty-six units and forty-eight hundredths, by four hundred seventy-five thousandth.

DIVISION.

148. Since the dividend is equal to the product of the divisor and quotient, it must have as many decimal figures as there are in both divisor and quotient. Hence,

The quotient must have as many decimal figures as the number of decimal figures in the dividend exceeds that in the divisor.

149. 1. Divide 1.264 by .4.

OPERATION.

.4)1.264

Ans. 3.16

1.264 divided by 4 gives .316; but, as the divisor is .4, the quotient must be ten times as large, and .316 multiplied by 10, by removing the decimal point one order to the right, gives 3.16. Therefore, the quotient of 1.264 divided by .4 is 3.16.

2. Divide .00115 by .05.

OPERATION.

05).00115

.023

.00115 divided by 5 gives .00023; but, as the divisor is .05, the quotient must be one hundred times as large, and .00023 multiplied by 100, by removing the point two orders to the right, gives .023. Therefore, the quotient of .00115 divided by .05 is .023.

Rule.—*Divide as in integers, and point off as many figures for decimals in the quotient as the number of decimal figures in the dividend exceeds that in the divisor, supplying the deficiency, if any, by prefixing ciphers; or,*

What is the dividend equal to? How many decimal figures must the quotient have? Explain the operation. Recite the Rule.

If the divisor is a decimal, divide by its numerator, and multiply by its denominator.

When the divisor is 10, 100, 1000, etc., remove the point to the left as many orders as there are ciphers in the divisor.

Examples.

Divide

3. 8.05 by .23. Ans. 35.
4. 8.05 by 2.3. Ans. 3.5.
5. 8.05 by 23. Ans. .35.
6. .805 by 2.3.
7. 80.5 by 2.3.
8. 9.065 by .049.
9. 1.31 by 100. Ans. .0131.
10. 15.625 by 2.5. Ans. 6.25.
11. 1.31 by 10.
12. 1.31 by 1000.

150. When the number of decimal figures in the divisor exceeds that in the dividend, *annex to the dividend as many ciphers for decimals, at least, as will make the number of decimal figures in both equal.*

13. Divide 805 by 2.3.

OPERATION.

$$2.3\overline{)805.0(350},\ \textit{Ans.}$$
$$\begin{array}{r} 69 \\ \hline 115 \\ 115 \\ \hline 0 \end{array}$$

14. Divide 1 by .13.

OPERATION.

$$.13\overline{)1.0000(7.69+}\ \textit{Ans.}$$
$$\begin{array}{r} 91 \\ \hline 90 \\ 78 \\ \hline 120 \\ 117 \\ \hline 3 \end{array}$$

Here, in Example 14, the division does not terminate. In such cases, after carrying it to a sufficient exactness, denote the incompleteness by annexing the sign.

How do you proceed when the divisor is 10, 100, 1000, etc.?

How do you proceed when the number of decimal figures in the *divisor exceeds that in* the dividend?

15. Divide 2.117 by .0073. Ans. 290.
16. Divide 8 by .002. Ans. 4000.
17. Divide 7.43 by .0079. Ans. 940.5063+.
18. Divide 89 by .003. Ans. 29666.66+.
19. Find the quotient of 80.3 divided by .22.
20. Find the quotient of 80.3 divided by 2.2.
Ans. 36.5.

21. If 3.5 acres produce 142.1 bushels of corn, what is the yield per acre? Ans. 40.6 bu.
22. How many yards of cotton cloth, at .125 dollars per yard, can be bought for $46.95?
Ans. 375.6 yd.
23. At .75 dollars a day, how many days must a boy work to earn $141? Ans. 188 days.
24. If you can walk 12.5 miles in one day, how many days will it take you to walk 102.5?
25. Bought 30 pounds of butter for $5.85; how much was that a pound? Ans. $0.195.
26. Divide one hundredth by one ten-thousandth.
Ans. 100.

REVIEW.

1. Add seven hundred forty-three thousandths, seven hundredths, five units and sixteen ten-thousandths.
Ans. 5.8146.
2. Add eighty-seven units and twenty-nine hundredths, seventy-three units and forty-seven thousandths, three thousand five units and one hundred six ten-thousandths,

---

REVIEW. — What is a Unit? (1.) What is the Unit of a Fraction? *Ans.* The unit or single thing divided into equal parts.

twenty-eight units and three hundredths, one unit and five thousandths. Ans. 3194.3826.

3. Reduce $5\frac{3}{16}$ to the decimal form. Ans. 5.1875.

4. Reduce $\frac{17}{25}$ and $\frac{13}{50}$ to decimals having a common denominator. Ans. .680 and .052.

5. From 100 subtract 101 thousandths.

Ans. 99.899.

6. From eighteen units and three hundred sixty-five thousandths, take eighteen hundred sixty-five ten-thousandths. Ans. 18.1785.

7. Change .68 to an equivalent common fraction.

Ans. $\frac{17}{25}$.

8. Change 5.25 to an equivalent common fraction.

Ans. $\frac{21}{4}$

9. Multiply one hundred units and thirty-seven hundredths by nine tenths. Ans. 90.333.

10. What common fraction is equal to the sum of .125 and .525? Ans. $\frac{13}{8}$.

11. Multiply .0235 by 8.08. Ans. .18988.

12. Divide .0625 by 2.5. Ans. .025.

13. At $1.25 a pair, how many pairs of shoes can be bought with $45? Ans. 36 pairs.

14. If an acre produces 16.25 bushels of wheat, how much will 3.05 acres produce?

15. If $18.5625 is paid for 148.5 yards of calico, how much is paid per yard? $.125.

16. If 3.5 cords of wood cost $22.4, how much will 4.25 cords cost? Ans. $27.20.

REVIEW. — What is a Decimal Fraction? (129.) What the Denominator of a Decimal Fraction? (131.) How many decimal figures has the product in Multiplication of Decimals? (147.) The *quotient in Division* of Decimals? (148.)

QUESTIONS FOR ANALYSIS.

1. At $2.25 per hundred weight, or cental, what is a load of wheat, weighing 2564 pounds, worth?

SOLUTION. — 2564 *pounds reduced to hundred weight, or centals, by pointing off two orders from the right, is* 25.64 *centals. At* $2.25 *per cental,* 25.64 *centals of wheat will be worth* 25.64 *times* $2.25, *or* $57.69. *Therefore, etc.*

2. What cost a cargo of corn weighing 156750 pounds, at $1.20 per cental? Ans. $1881.

3. How much must be paid for 11023 pounds of corn, at $1.05 per cental? Ans. 115.74+.

4. At $1.20 per hundred weight, what is the freight on 6545 pounds of glass ware? Ans. $78.54.

5. At $8 per thousand, what cost 19340 bricks?

SOLUTION. — 19340 *reduced to thousands, by pointing off three orders from the right, is* 19.34 *thousands. At* $8 *per thousand,* 19.34 *thousands will cost* 19.34 *times* $8, *or* $154.72. *Therefore, etc.*

6. What cost 42555 feet of boards, at $40 per thousand?

7. How much must be paid for 105750 feet of lumber, at $14 a thousand? Ans. $1480.50.

8. At $5 per ton of 2000 pounds, what cost 5460 pounds of coal?

SOLUTION. — 5460 *pounds is* 5.46 *thousand pounds, and since* 2 *thousand pounds make a ton, there must be half as many tons as there are thousand pounds, or* 2.73 *tons. At* $5 *per ton,* 2.73 *tons of coal will cost* 2.73 *times* $5, *or* $13.65. *Therefore, etc.*

9. At $25 per ton, what cost 3640 pounds of hay?

10. How much must be paid for 4260 pounds of plaster, at $8 per ton? Ans. $17.04.

WEIGHTS AND MEASURES.

AVOIRDUPOIS WEIGHT.

151. Avoirdupois Weight is used for nearly all articles estimated by weight, except gold, silver, and jewels.

Table.

16 drams (dr.)	are	1 ounce,	oz.
16 ounces,		1 pound,	lb.
100 pounds,		1 hundred-weight,	cwt.
20 hundred-weight,		1 ton,	T.

Also, 25 pounds are 1 quarter; 2000 pounds 1 ton.

Formerly 112 pounds were reckoned a hundred-weight, and 2240 pounds a ton, but seldom now, except for coal at the mines, or goods at the United States custom-houses. The dram is used by silk manufacturers.

For what is Avoirdupois Weight used? Recite the Table. How *many pounds were formerly* a hundred-weight? A ton?

MENTAL EXERCISES.

1. How many drams in 2 ounces? In 4 ounces?
2. How many ounces in 3 pounds? In 5 pounds?
3. How many pounds in 3 quarters? In 5 quarters?
4. How many quarters in 5 hundred-weight?
5. How many hundred-weight in 2 tons? In 5 tons?
6. How many ounces in 32 drams? In 48 drams?
7. How many pounds in 64 ounces? In 48 ounces?
8. How many pounds in 4 hundred-weight? In 10 hundred-weight?
9. How many hundred-weight in 8 quarters? In 24 quarters?
10. How many tons in 40 hundred-weight? In 80 hundred-weight?

TROY WEIGHT.

152 Troy Weight is used for gold, silver, and jewels.

Table.

24 grains (gr.)	are	1 pennyweight,	pwt.
20 pennyweights,		1 ounce,	oz.
12 ounces,		1 pound,	lb.

APOTHECARIES, in mixing medicines, use the *pound*, *ounce* (℥), and *grain*, of this weight; but divide the ounce into 8 *drams* (ʒ), each equal to three *scruples* (℈), each scruple being equal to 20 *grains*.

A pound Avoirdupois is equal to 7000 grains Troy, and a pound Troy to 5760 grains.

For what is Troy Weight used? Recite the Table. What denominations of Troy Weight do apothecaries use? How is the ounce divided by them? How many grains Troy are equal to a pound Avoirdupois? To a pound Troy?

MENTAL EXERCISES.

1. How many grains in 2 pennyweights? In 3 pennyweights? In 4 pennyweights?

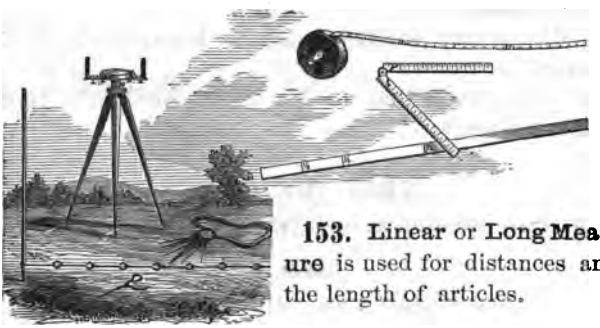
2. How many pennyweights in 3 ounces? In 5 ounces? In 4 ounces?

3. How many ounces in 4 pounds? In 5 pounds?

4. How many pennyweights in 48 grains?

5. How many pounds in 32 ounces? In 48 ounces?

LINEAR MEASURE.



153. **Linear** or **Long Measure** is used for distances and the length of articles.

Table.

12 inches (in.)	are 1 foot,	ft.
3 feet,	1 yard,	yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet,	1 rod,	rd.
320 rods,	1 mile,	m.

Also, 4 inches are 1 hand; 6 feet 1 fathom; 40 rods 1 furlong; 8 furlongs 1 mile; 3 miles 1 league.

For what is Linear Measure used? Recite the Table. How many inches make 1 hand? How many feet 1 fathom? How many *miles* 1 league?

In measuring *cloth* and other woven fabrics, the yard is divided into *halves, quarters, eighths,* and *sixteenths.* Formerly the sixteenth, or $2\frac{1}{4}$ inches, was called a *nail.*

MENTAL EXERCISES.

1. How many inches in 2 feet? In 7 feet?
2. How many feet in 4 yards? In 11 yards?
3. How many yards in 2 rods? In 4 rods?

SOLUTION. — *Since there are* $5\frac{1}{2}$ *yards in* 1 *rod, there are* 2 *times* $5\frac{1}{2}$ *yards, or* 2 *times* 5 *yards and* 2 *times* $\frac{1}{2}$ *yard in* 2 *rods;* 2 *times* 5 *yards are* 10 *yards, and* 2 *times* $\frac{1}{2}$ *yard is* 1 *yard;* 10 *yards and* 1 *yard are* 11 *yards. Therefore, there are* 11 *yards in* 2 *rods.*

4. How many rods in 2 furlongs? In 3 furlongs?
5. How many furlongs in 6 miles? In 9 miles?
6. How many yards in 18 feet? In 30 feet?
7. How many rods in 11 yards?

SOLUTION. — *Since there is in* $5\frac{1}{2}$ *yards, or in* 11 *half yards,* 1 *rod, there are as many rods in* 11 *yards, or* 22 *half yards, as* 11 *half yards are contained times in* 22 *half yards, which are* 2. *Therefore, there are* 2 *rods in* 11 *yards.*

8. How many furlongs in 80 rods? In 120 rods?
9. How many miles in 16 furlongs? In 24 furlongs?
10. How many inches are 2 feet 6 inches? How many furlongs are 3 miles 5 furlongs?
11. How many feet in 30 inches? How many miles in 31 furlongs?
12. How many quarters are 7 yards 2 quarters? How many eighths are 5 yards 5 eighths?
13. How many yards in 30 quarters? How many yards in 43 eighths?

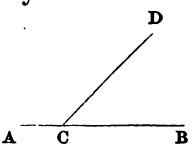
How is the yard divided in measuring cloth and other woven fabrics?

SURFACE MEASURE.

154. A **Surface** is that which has only length and breadth.

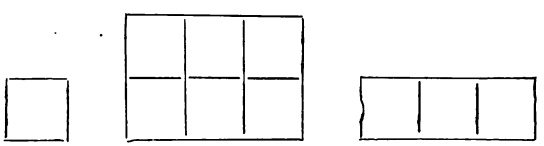
155. The **Area** of a figure is its quantity of surface.

156. An **Angle** is the difference in direction of two straight lines, which meet at a point. Thus, ACD and DCB are angles.



157. A **Square** is a surface having four equal sides and four equal angles.

A **Rectangle** is any surface having four sides and four equal angles.



158. The area of a square, each of whose sides is 1 foot, is 1 square foot.

The area of a rectangle, whose length is 3 feet, and whose width is 2 feet, is 6 square feet; for it is composed of 2 rows of 3 square feet, or of 2 times 3 square feet. Hence,

The area of a rectangle is equal to the product of the length by the breadth.

What is a Surface? The Area of a figure? An Angle? A *Square? A Rectangle?* To what is the area of a rectangle equal?

159. Surface or Square Measure is used in measuring surfaces.

Table.

144 square inches (sq. in.) are	1 square foot,	sq. ft.
9 square feet,	1 square yard,	sq. yd.
$30\frac{1}{4}$ square yards,	1 square rod or perch,	P
160 square rods or perches,	1 acre,	A.
640 acres,	1 square mile,	M.

Land is usually measured by Gunter's chain, which is 4 rods, or 66 feet, long, and divided into 100 links, each $7\frac{92}{100}$ inches long. 1 *square chain* is 16 square rods, and 10 square chains are 1 acre. 640 **acres are one** *section* of land.

A *rood* (R.) equals 40 square rods or perches.

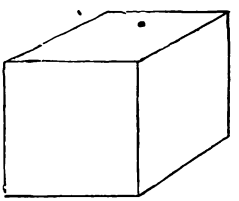
MENTAL EXERCISES.

1. How many square inches in 1 square foot?
2. How many square feet in 5 square yards?
3. How many square yards in 1 square rod? In 4 square rods?
4. How many square rods in 2 roods? In 3 roods?
5. How many roods in 5 acres? In 10 acres?
6. How many acres make 1 square mile?
7. How many square yards in 45 square feet? In 81 square feet?
8. How many square rods in 121 square yards?
9. How many roods in 80 square rods? In 120 square rods?
10. How many acres in 16 roods? In 28 roods?

How is the Area of a Rectangle found? For what is Surface Measure used? Recite the Table. How is land usually measured?

SOLID MEASURE.

160. A **Solid,** or **Volume,** is that which has length, breadth, and hight or depth.

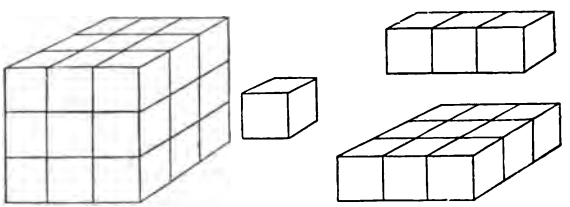


A Cube.

161. A **Cube** is a volume bounded by six square and equal sides or faces.

If a cube is 1 yard or 3 feet long, 1 yard or 3 feet wide, and 1 yard or 3 feet high, it is a cubic or solid yard.

162. A **Rectangular Solid, or Volume,** is any volume bounded by rectangular faces.



In the large cube are represented 3 tiers of small cubes, of 3 rows each, and 3 in a row. If each of these small cubes is supposed to be one cubic foot, then in the large cube there evidently are in one of the equal tiers 3 times 3 cubic feet, or 9 cubic feet, and in the 3 tiers, 3 times 9 cubic feet, or 27 cubic feet, or as many as the product of the length, breadth, and hight or depth. Hence,

The contents of a rectangular solid, or volume, are equal to the product of the length, breadth, and hight or depth.

What is a Solid, or Volume? A Cube? A Rectangular Solid? *To what are the contents* of a Rectangular Solid equal?

163. Solid or Cubic Measure is used in measuring bodies, or things, having length, breadth, and hight or depth.

Table.

1728 cubic inches (cu. in) are	1 cubic foot,	cu. ft.
27 cubic feet,	1 cubic yard,	cu. yd.
128 cubic feet,	1 cord,	C.

A pile of wood 8 feet long, 4 feet wide, and 4 feet high, is a *cord*. A *cord foot* (c. f.) is 1 foot in length of this pile, or 16 cubic feet.

1 Cord. 1 c. f.

A *perch* of masonry, or of building stone, is $24\frac{3}{4}$ cubic feet.

A *ton* of timber is usually estimated at 40 cubic feet, but as usually measured is about 50 cubic feet.

MENTAL EXERCISES.

1. How many cubic inches in 1 cubic foot?
2. How many cubic feet in 2 cubic yards?
3. How many cubic feet in 2 tons? In 3 tons?
4. How many cubic feet in a cord?
5. How many cubic yards in 54 cubic feet? In 81 cubic feet?
6. How many tons in 80 cubic feet? In 120 cubic feet?
7. How many cord feet in 7 cords? In 9 cords?
8. How many cords in 56 cord feet? In 72 cord feet?
9. How many cubic feet in 3 cubic yards? In 4 cubic yards?

For what is Solid Measure used? Recite the Table. What is a cord of wood? A cord foot? A perch of stone?

LIQUID MEASURE.

164. Liquid Measure is used in measuring all kinds of liquids.

Table.

4 gills (gi.)	are	1 pint,	pt.
2 pints,		1 quart,	qt.
4 quarts,		1 gallon,	gal.

The *liquid gallon* contains 231 cubic inches. Formerly ale, beer, porter, and milk were sold by what was called *Beer Measure*, and of which the gallon contained 282 cubic inches.

In some States 31½ gallons, and in others, 28 to 32 gallons, are considered a *barrel*.

A *hogshead* regarded as a measure is 63 gallons.

For what is Liquid Measure used? Recite the Table. How many cubic inches in a liquid gallon? How were beer and milk formerly sold? How many gallons in a barrel? In a hogshead?

MENTAL EXERCISES.

1. How many gills in 4 pints? In 8 pints?
2. How many pints in 8 quarts? In 10 quarts?
3. How many quarts in 5 gallons? In 9 gallons?
4. How many gallons in 1 hogshead?
5. How many pints in 16 gills? In 32 gills?
6. How many gallons in 20 quarts? In 36 quarts?
7. How many hogsheads in 63 gallons?

DRY MEASURE.

165. Dry Measure is used in measuring such dry articles as grain, fruit, roots, coal, etc.

Table.

2 pints (pt.)	are 1 quart,	qt.
8 quarts,	1 peck,	pk.
4 pecks,	1 bushel,	bu.

A gallon, or 4 quarts, Dry Measure, contains $268\frac{1}{5}$ cubic inches, and a bushel, $2150\frac{42}{100}$ cubic inches. A chaldron, used in measuring coal, contains 36 bushels.

MENTAL EXERCISES.

1. How many pints in 8 quarts? In 10 quarts?
2. How many quarts in 4 pecks? In 6 pecks?
3. How many pecks in 4 bushels? In 8 bushels?
4. How many bushels in 2 chaldrons?
5. How many quarts in 32 pints? In 40 pints?
6. How many bushels in 16 pecks? In 32 pecks?
7. How many chaldrons in 72 bushels?

What is Dry Measure? Recite the Table. How many cubic in a gallon, Dry Measure?

TIME.

166. **Time** is a measured portion of duration.

Table.

60 seconds (sec.)	are 1 minute,	m.
60 minutes,	1 hour.	h.
24 hours,	1 day,	d.
365 days,	1 common year,	c. y.
366 days,	1 leap year,	l. y.

Also,

7 days	are 1 week,	w.
12 calendar months (mo.)	1 year,	y.
100 years,	1 century,	C.

167. The **Calendar Months**, their names, order, and number of days, are as follows: —

January,	1st	mo.	31 days.	July,	7th	mo.	31 days.
February,	2d	"	28 or 29.	August,	8th	"	31 "
March,	3d	"	31 days.	September,	9th	"	30 "
April,	4th	"	30 "	October,	10th	"	31 "
May,	5th	"	31 "	November,	11th	"	30 "
June,	6th	"	30 "	December,	12th	"	31 "

The exact length of an average year being about 365¼ days, the ¼ day in 4 years makes an additional day. This day, leap year, makes February have 29 days.

168. Leap year may be known by being such that its number can be exactly divided by 4. Thus, 1864, 1868, and 1872 are leap years, and have 366 days.

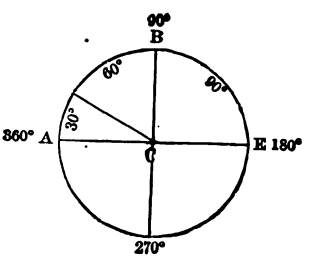
What is time? Recite the Table. Give the names of the calendar months. Length of an average year. How may leap year be *known?*

MENTAL EXERCISES.

1. How many seconds in 3 minutes?
2. How many minutes in 2 hours? In 3 hours?
3. How many hours in 2 days? In 3 days?
4. How many days in a common year?
5. How many days in a leap year?
6. How many minutes in 120 seconds?
7. How many minutes in 2 hours? In 4 hours?
8. How many hours in 120 minutes?
9. How many days in 48 hours? In 72 hours?
10. How many years in 365 days?

CIRCULAR MEASURE.

169. A **Circle** is a plane figure, bounded by a curved line, all the points of which are equally distant from a point within, called the *center*.



170. The **Circumference** of a circle is its entire bounding line.

171. An **Arc** of a circle is any part of the circumference, as AB, or AE.

172. A **Diameter** of a circle is any straight line drawn through the center, and terminated by the circumference, as AE.

173. The circumference of every circle is supposed to be divided into 360 equal parts, called **Degrees.**

What is a Circle? The Circumference of a circle? What is an Arc of a circle? A Diameter? How is every circle divided?

A **Quadrant** is a fourth of a circumference, or an arc of 90 degrees, as AB.

174. The **Angle** ACB, whose two lines meet at the center C, is measured by the arc of the circle drawn around that center, included between the opening of those lines. Thus, ACB is an angle of 90 degrees.

A **Right Angle** is an angle of 90 degrees.

175. **Circular Measure** is used in measuring angles, and arcs of circles.

Table.

60 seconds (″)	are 1 minute, ′.
60 minutes,	1 degree, °.
360 degrees,	1 circum. C.

Also, 30 degrees are 1 sign (S.); 12 signs 1 circumference.

MENTAL EXERCISES.

1. How many seconds in 2 minutes? In 3 minutes?
2. How many minutes in 2 degrees? In 3 degrees?
3. How many degrees in 2 signs? In 4 signs?
4. How many signs in 4 circles? In 5 circles?
5. How many minutes in 120 seconds? In 240 seconds?
6. How many degrees in 180 minutes? In 120 minutes?
7. How many signs in 60 degrees? In 90 degrees?
8. How many circles in 360 degrees?
9. How many degrees in 8 signs? In 7 signs?

What is a Quadrant? A Right Angle? What is Circular Measure? *Recite the* Table.

7. How many signs in 120 degrees? In 150 degrees?
8. How many circles in 720 degrees?

Miscellaneous Tables.

176. Counting.

12 units	are 1	dozen.
12 dozen,	1	gross.
12 gross,	1	great gross.
20 units,	1	score.

177. Paper.

24 sheets	are 1	quire.
20 quires,	1	ream.
2 reams,	1	bundle.
5 bundles,	1	bale.

178. Capacity.

56 pounds of rye or corn	are	1	bushel.
60 pounds of wheat,		1	bushel.
60 pounds of beans or peas,		1	bushel.
60 pounds of potatoes,		1	bushel.
60 pounds of clover-seed,		1	bushel.
45 pounds of timothy-seed,		1	bushel.
100 pounds of dry fish,		1	quintal.
100 pounds of grain or flour,		1	cental.
196 pounds of flour,		1	barrel.
200 pounds of beef or pork,		1	barrel.

Indian meal, or *cracked corn*, is usually estimated at 50 pounds to a bushel, and *wheat bran* at 20 pounds to a bushel.

Recite the Table for Counting. For Paper. For Capacity.

MENTAL EXERCISES.

1. How many single things in 2 dozen?
2. How many single things in 2 scores?
3. How many sheets in 2 quires? In 3 quires?
4. How many quires in 4 reams? In 5 reams?
5. How many pounds of wheat in 2 bushels?
6. How many pounds of fish in 5 quintals?
7. How many pounds of pork in 3 barrels?
8. How many bushels in 120 pounds of wheat?
9. How many quintals of fish in 500 pounds?
10. How many barrels of pork in 600 pounds?

REDUCTION.

179. 1. How many inches in 3 yards?

SOLUTION. — *Since there are* 3 *feet in* 1 *yard, in* 3 *yards there are* 3 *times* 3 *feet, which are* 9 *feet; and since there are* 12 *inches in* 1 *foot, in* 9 *feet there are* 9 *times* 12 *inches, which are* 108 *inches. Therefore, there are in* 3 *yards* 108 *inches.*

2. How many gills in 2 gallons?
3. How many yards in 108 inches?

SOLUTION. — *Since in* 12 *inches there is* 1 *foot, in* 108 *inches there are as many feet as* 12 *inches are contained times in* 108 *inches, or* 9; *and since in* 3 *feet there is* 1 *yard, in* 9 *feet there are as many yards as* 3 *feet are contained times in* 9 *feet, or* 3. *Therefore, in* 108 *inches there are* 3 *yards.*

5. How many gallons in 64 gills?
6. How many yards in 72 inches?

REVIEW. — What is a Surface? A Rectangle? A Solid or Volume? A Circle?

180. **Reduction** is the process of changing one number into another of a different denomination, but of equal value.

REDUCTION DESCENDING.

181. **Reduction Descending** is the changing of one number into another of a lower denomination of equal value.

WRITTEN EXERCISES.

182. 1. Reduce 18 T. 5 cwt. 3 qr. to quarters.

OPERATION.

18 T. 5 cwt. 3 qr.

20

360

5

365 cwt.

4

1460

3

1463 qr.

Since in 1 ton there are 20 hundred, in 18 tons there are 18 times 20 hundred, or 360 hundred, and 5 hundred added make 365 hundred, in 18 T. 5 cwt.

Since in 1 hundred there are 4 quarters, in 365 hundred there are 365 times 4 quarters, or 1460 quarters, and 3 quarters added make 1463 quarters.

Therefore, 18 T. 5 cwt. 3 qr. reduced to quarters is 1463 quarters.

Rule.—*Multiply the number of the highest denomination given by the number required of the next lower denomination to make one of that higher, and to the product add the number, if any, of the lower denomination.*

Proceed in like manner till the whole is reduced to the required denomination.

What is Reduction? What is Reduction Descending? Recite the Rule. How do you multiply by ½? *Ans.* By taking half of the multiplicand, or by dividing by 2.

Examples.

2. Reduce 27 cwt. 2 qr. 22 lb. to pounds.

OPERATION.

27 *cwt.* 2 *qr.* 22 *lb*

4

108

2

110 *qr.*

25

550

220

2750

22

2772 *lb.*

3. Reduce 5 fur. 12 rd. 4 yd. to yards.

OPERATION.

5 *fur.* 12 *rd.* 4 *yd.*

40

200

12

212 *rd.*

$5\frac{1}{2}$

1060

106

1166

4

1170 *yd.*

It will be observed, in the second operation, that $5\frac{1}{2}$ times 212 is the same as 5 times 212, and $\frac{1}{2}$ of 212; 5 times 212 is 1060, and $\frac{1}{2}$ of 212 is 106; 1060 and 106 are 1166.

4. Reduce 1 lb. 3 oz. 5 pwt. to pennyweights.
5. Reduce 37 A. 3 R. 12 P. to square yards.
6. In 7 cu. yd. 20 cu. ft. how many cubic inches? *
7. In 4 hogsheads how many quarts?
8. Reduce 45 bu. 3 pk. 5 quarts to quarts.
9. In 10 h. 17 m. 18 s. how many seconds?
10. In 3° 4′ 16″ how many seconds?
11. In 2 T. 9 cwt. how many pounds?

* *For answers, see* corresponding examples in Reduction Ascending.

12. Reduce 84 gal. 2 qt. 1 pt. to gills.
13. How many ounces in 12 cwt. 1 qr. 0 lb.?
14. How many minutes in 3 wk. 5 d. 10 h.?
15. How many rods in 32 m. 0 fur. 25 rd.?
16. How many square rods in 14 A. 1 R. 35 P.?
17. How many cord feet in 96 cords?
18. How many seconds in 45° 56′ 30″?

183. The preceding rule, in its application to United States Money, since the multipliers will be 10, 100, or 1000, admits of an abridged operation. Thus: —

To reduce cents to mills, annex ONE *cipher.*

To reduce dollars to cents, annex TWO *ciphers, and remove the dollar mark and decimal point.*

To reduce dollars to mills, annex THREE *ciphers, and remove the dollar mark and decimal point.*

To reduce dollars and cents to cents, or dollars, cents, and mills to mills, remove the dollar mark and decimal point.

19. Reduce $98 to cents.
20. Reduce 168 cents to mills.
21. Reduce $106 to mills.
22. In $37.19 how many cents?
23. How many cents in $136.05?
24. How many mills in $1325?
25. How many mills in 69 cents?
26. How many mills in $215.09?

REDUCTION ASCENDING.

184. **Reduction Ascending** is the changing of one number into another of a higher denomination of equal value.

How can you reduce dollars to cents? Cents to mills? Dollars to mills? Dollars and cents to cents? Dollars, cents, and mills to mills? What is Reduction Ascending?

WRITTEN EXERCISES.

185. 1. Reduce 1463 quarters to tons.

OPERATION.

4)1463 qr.

20)365 3 qr.

18 T. 5 cwt.

Ans. 18 T. 5 cwt. 3 qr.

Since in 4 quarters there is 1 hundred, in 1463 quarters there are as many hundred as 4 quarters are contained times in 1463 quarters, which are 365, and 3 quarters remaining.

Since in 20 hundred there is 1 ton, in 365 hundred there are as many tons as 20 hundred are contained times in 365 hundred, which are 18, and 5 hundred remaining.

Therefore, there are in 1463 quarters, 18 T. 5 cwt. 3 qr.

Rule.—*Divide the given number by the number of its denomination required to make one of the next higher, and reserve the remainder, if any.*

Proceed in like manner with the quotient, and so continue till the whole is reduced to the required denomination.

The number of the required denomination, with the several remainders, if any, will be the answer.

Examples.

2. Reduce 2772 pounds to hundreds.

OPERATION.

25)2772 lb.

4)110 . . 22 lb.

27 . . 2 qr.

Ans. 27 cwt. 2 qr. 22 lb.

3. Reduce 1170 yards to furlongs.

OPERATION.

$5\frac{1}{2}$)1170 yd.

2 2

11)2340 hlf. yd.

40)212 . . 8 hlf. yd. = 4 yd.

5 . . 12 rd.

Ans. 5 fur. 12 rd. 4 yd.

°Recite the Rule.

In the second operation, to divide 1170 by $5\frac{1}{2}$, we first reduce both numbers to halves, by multiplying by 2, and have 2340 *halves* to be divided by 11 *halves*, which gives 212, and a remainder of 8 halves = 4.

4. Reduce 305 pennyweights to pounds.
5. Reduce 183073 square yards to acres.
6. Reduce 361152 cubic inches to cubic yards.
7. Reduce 1008 quarts to hogsheads.
8. Reduce 1469 quarts to bushels.
9. Reduce 37038 seconds to hours.
10. Reduce 11056 seconds to degrees.
11. In 4000 pounds how many tons?
12. In 2708 gills how many hogsheads?
13. In 19600 ounces how many hundreds?
14. In 38040 minutes how many weeks?
15. In 10265 rods how many miles?
16. How many acres in 2315 square rods?
17. How many cords in 768 cord feet?
18. How many degrees in 165390 seconds?

186. The preceding rule, in its application to United States money, since the divisors will be 10, 100, or 1000, admits of an abridged operation.

To reduce cents to dollars, point off TWO *figures from the right, and prefix the dollar mark; to reduce mills to dollars, point off* THREE *figures, and prefix the dollar mark.*

To reduce mills of two or more places of figures to cents, consider the figure on the right to express mills, and the other cents.

19. Reduce 9800 cents to dollars.
20. Reduce 1680 mills to cents.

How can you reduce cents to dollars? Mills to dollars? Mills to cents?

21. Reduce 106000 mills to dollars.
22. In 3719 cents how many dollars?
23. How many dollars in 13605 cents?
24. How many dollars in 1325000 mills?
25. How many cents in 690 mills?
26. How many dollars in 215090 mills?

REVIEW.

1. How many pounds in 7 T. 18 cwt.?
2. How many tons in 15800 pounds?
3. How many grains in 2 lb. 11 oz. 20 gr.?
4. How many pounds in 16820 grains?
5. How many quarts in 32 hogsheads?
6. How many hogsheads in 8064 quarts?
7. How many pints in 27 hhds. 18 gal. 2 qts.?
8. In 13756 pints how many hogsheads?
9. How many quarts in 17 bu. 1 pk. 4 qt.?
10. How many bushels in 556 quarts?
11. In 12 rd. 5 yd. 2 ft. how many feet?
12. In 215 feet how many rods?
13. Reduce 3 A. 3 R. 20 P. to square yards
14. Reduce 18755 square yards to acres.
15. Reduce 3 cu. yd. 9 cu. ft. 1480 cu. in. to cu. in.
16. Reduce 157000 cubic inches to cubic yards.
17. Reduce 506 d. 22 h. 40 min. to minutes.
18. Reduce 730000 minutes to days.
19. How many minutes in one common year?
20. How many years in 525600 minutes?

How is Reduction Descending performed? *Ans.* By Multiplication. How is Reduction Ascending performed? *Ans.* By Division. How then do their operations differ? How is each kind of Reduction proved? *Ans.* Each by the process of the other.

21. Reduce 14° 5′ to seconds.

22. Reduce 50700 seconds to degrees.

23. In 5 reams of paper how many sheets?

24. In 2400 sheets of paper how many reams?

25. Reduce $46.05 to mills.

26. In 46050 mills how many dollars?

27. In 20 eagles how many cents?

28. In 20000 cents how many eagles?

29. In $31.99 how many cents?

30. In 3199 cents how many dollars?

31. In 573 yd. 1 qr. 1 na. how many nails?

32. In 9173 nails how many yards?

33. In 3 miles how many inches?

34. In 190080 inches how many miles?

35. In 57935 mills how many dollars?

36. In $57.935 how many mills?

37. In a pile of wood 96 feet long, 4 feet high, and 4 feet wide, how many cords? Ans. 12 cords.

38. How many square feet in a board 20 feet long and 2 feet wide? Ans. 40 square feet.

39. What cost 1 quarter-hundred of beef, at 9 cents a pound?

SOLUTION. — *Since* 1 *quarter hundred is* 25 *pounds*, 1 *quarter-hundred of beef, at* 9 *cents a pound, will cost* 25 *times* 9 *cents, which are* 225 *cents, or* $2.25. *Therefore, it will cost* $2.25.

40. What cost a barrel of flour, at 6 cents a pound?

41. What is the value of a rectangular lot, 40 rods long, and 20 rods wide, at 50 cents a square rod?

42. In a block of stone, 6 feet long and 2 feet square, how many cubic feet? Ans. 24 cu. ft.

REVIEW. — Recite the Table of United States Money. (69.) Of Avoirdupois Weight. (151.) Of Troy Weight. (152.) Of Linear Measure. *(153.)*

43. How many grains in 15 silver spoons, each weighing 6 pwt. 15 gr.? Ans 2385.

44. What is the cost of levelling a plat of ground, 20 rods long and 20 rods wide, or 20 rods square, at 25 cents a square rod? Ans. $100.

45. What is the value of a pile of wood 20 feet long, 6 feet high, and 4 feet wide, at 50 cents a cord foot?

Ans. $15.

46. What will it cost to pave with marble the floor of a room, 4 yards square, at $1.25 per square foot?

47. How many hundred-weight of sugar, at 15 cents a pound, can be bought for $20.40?

SOLUTION. — *There will be as many pounds as* 15 *cents are contained times in* $20.40, *or in* 2040 *cents, which are* 136; *and as many hundred-weight as are in* 136 *pounds, or* 1 *cwt.* 1 *qr.* 11 *lb. Therefore, there can be bought* 1 *cwt.* 1 *qr.* 11 *lb.*

48. How many hogsheads of molasses, at 12 cents a quart, can be bought for $75.12?

Ans. 2 hhd. 30 gal. 2 qt.

49. If the heart beat 70 times in a minute, how many days will be required for it to beat 705600 times?

Ans. 7 days.

50. If a ship can sail at the rate of 8 miles an hour, how many weeks will be required for it to sail 4448 miles?

Ans. 3 wk. 2 d. 4 h.

51. If a comet move through 10° in one day, how many signs will it move through in 14 days?

Ans. 4 S. 20°

52. If 9 candles can be made from 1 pound of tallow, how many dozen can be made from 60 pounds?

Ans. 45 dozen.

REVIEW. — Recite the Table of Square Measure. (159.) Of Solid Measure. (163.) Of Time Measure. (166.) Of Circular Measure. (175.)

53. How many minutes in an average year of 365 days, 6 hours? Ans 525960 m.

54. How many hours from the 12th of January to the 24th of the next April in a common year?

Ans. 2448 h.

SOLUTION. — *There will be as many days as are between the dates;* 31 *days in January less* 12 *days* = 19 *days;* 19 *days in Jan.* + 28 *days in Feb.* + 31 *days in Mar.* + 24 *days in Apr.* = 102 *days; and since there are* 24 *hours in* 1 *day, there will be* 102 *times* 24 *hours, or* 2448 *hours. Therefore, there will be* 2448 *hours.*

55. In leap year how many hours from the 8th of January to the 12th of December? Ans. 8136.

56. What is the value of 2 R. 10 P. of land, at 6 cents per square foot?

57. What will 3 T. 5 cwt. 20 lb. of iron cost, at 3 cents a pound?

58. At 3 cents a pound, how many tons of iron can be bought for $195.60?

59. How wide must be the floor of a rectangular room, whose length is 8 yards, to require 48 square yards to cover it? Ans. 6 yards.

Here, the 48 is evidently the product of the length and the width, and if we divide the 48 by the length given, or 8, we have the width required.

60. How long must a piece of land be, whose width i 20 rods, to contain just 5 acres? Ans. 40 rods.

61. What cost 645 pounds of potatoes, at 20 cents a peck?

REVIEW. — Recite the Table of Liquid Measure. (164.) Dry Measure. (165.)

62. How many pounds of potatoes, at 20 cents a peck, can be bought for $8.60?

63. How many days will it take to count a million of dollars, at the rate of a hundred a minute?

Ans. 6 d. 22 h. 40 m.

64. How many cords in a pile of wood 9 feet long, 8 feet high, and 4 feet wide? Ans. 2 c. 2 c. f.

65. A pile of wood is 4 feet wide, and 4 feet high; how long must it be to contain 2 cords? Ans. 16 feet.

Here, the product of the length, width, and hight is equal to the number of cubic feet in 2 cords, or 128 cu. ft. × 2, which is 256 cu. ft.; and if we divide the 256 by the product of the width and hight, which is 16, we have the length required.

66. How high must a range of wood, 20 feet in length and 4 feet wide, be piled to make 5 cords?

Ans. 8 feet.

67. How many times will a wheel 18 ft. 4 in. in circumference turn, in going 100 miles?

Ans. 28800 times.

68. How many loaves of bread can be made from 1 barrel and 4 pounds of flour, allowing 10 ounces to a loaf? Ans. 320.

69. How many shingles will be required to cover the side of a roof 30 feet by 20 feet, allowing that 1 thousand just cover 100 square feet? Ans. 6 thousand.

70. How many days of 10 hours each may be saved for study, in 10 years of 365 days each, by rising 30 minutes earlier each day? Ans. 182 d. 5 h.

71. How much will 3 cords 7 cord feet of oak wood cost, at $1 per cord foot?

72. How many weeks are there in 1000000 seconds?

Ans. 1 wk. 4 d. 13 h. 46 min. 40 sec.

COMPOUND NUMBERS.

187. A Simple Number is of a a single denomination; as, 5 dollars, 8 yards, 4 books.

188. A Compound Number is of two or more denominations of the same general nature; as, 5 dollars 50 cents; 4 yards 2 feet 6 inches; 12 hours 20 minutes.

The methods of adding, subtracting, multiplying, and dividing simple numbers apply, with some modifications, to compound numbers.

ADDITION.

189. 1. Let it be required to find the sum of 5 cwt. 2 qr. 10 lb., 2 cwt. 2 qr. 17 lb., and 10 cwt. 3 qr. 21 lb.

OPERATION.

	cwt.	qr.	lb.
	5	2	10
	2	2	17
	10	3	21
Ans.	19	0	23

For convenience, we write the numbers so that denominations of the same kind stand in the same column, and begin at the right to add.

21, 17, and 10 lb. are 48 lb., which, reduced, gives 1 qr. 23 lb.; we write the 23 lb. and add the 1 qr. in with quarters.

1, 3, 2, and 2 qr. are 8 qr., which, reduced, gives 2 cwt. 0 qr.; we write the 0, and add the 2 cwt. with the hundred-weights.

2, 10, 2, and 5 cwt. are 19 cwt., which we write.

Therefore the sum required is 19 cwt. 0 qr. 23 lb.

Rule.— *Write the numbers so that units of the same denomination shall stand in the same column.*

Beginning at the right, add each column, and reduce the sum to the next higher denomination; write the remainder, if any, and add the quotient with the next column.

What is a Simple Number? A Compound Number? How do you write the numbers? How do you add? What is the Rule?

PROOF. — The same as in simple numbers.

Examples.

(2.)

cwt.	qr.	lb.	oz.
2	1	0	10
5	3	16	15
1	0	1	2
9	0	18	11

(3.)

lb.	oz.	pwt.	gr.
21	8	19	20
10	4	11	16
9	13	0	5
42	2	11	17

(4.)

rd.	yd.	ft.	in.
1	3	2	1
16	4	2	11
20	2	1	10
39	0	0	10

(5.)

M.	fur.	rd.	yd.
6	5	16	3
5	0	23	8
10	3	19	2

(6.)

A.	R.	P.
27	2	31
35	3	24
22	1	17
45	0	29
131	0	21

(7.)

A.	R.	P.
41	2	28
27	1	31
51	0	19
42	1	25

(8.)

C.	cu. ft.
5	100
3	27
3	13
7	20
19	32

(9.)

cu. yd.	cu. ft.	cu. in.
6	21	1220
4	10	1531
9	11	160
	9	30

(10.)

T.	cu. ft.
15	25
7	16
	24
3	15
27	0

What is the Proof?

(11.)

hhd.	gal.	qt.	pt.	gi.
3	16	2	1	1
4	20	3	1	3
	41	1	1	3
6	0	0	1	1
14	16	1	0	0

(12.)

hhd.	gal.	qt.	pt.	gi.
5	0	3	1	2
8	50	2	1	0
6	25	1	0	3
	46	2	1	2

(13.)

bu.	pk.	pt.	qt.
23	3	5	1
10	2	6	1
16	1	3	1
20	1	2	0
70	1	1	1

(14.)

bu.	pk.	pt.	qt.
31	3	6	1
25	1	4	1
32	2	2	1
4	3	0	1

(15.)

d.	h.	m.	s.
5	13	39	42
6	20	29	45
4	22	19	33
4	17	59	59
22	2	28	59

(16.)

d.	h.	m.	s.
4	11	39	28
7	23	19	19
5	21	40	29
2	13	10	32

(17.)

s.	°	′	″
2	10	25	40
1	11	55	0
1	4	3	10
1	0	15	23
5	26	39	13

(18.)

s.	°	′	″
4	14	16	25
1	10	24	0
	12	0	17
3	9	10	8

19. What is the sum of 1 T. 5 cwt. 3 qr. 10 lb., 7 cwt. 20 lb., and 5 T. 3 qr.? Ans. 6 T. 13 cwt. 3 qr. 5 lb.

20. What is the sum of 4 yd. 2 ft. 6 in., and 5 yd. 1 ft. 11 inches? Ans. 10 yd. 1 ft. 5 in.

21. What is the sum of 9 oz. 18 pwt. 11 gr., 4 oz. 19 pwt. 20 gr., and 1 oz. 23 gr.?

Ans. 1 lb. 3 oz. 19 pwt. 6 gr.

22. Two men leave the same place and travel, the one north 65 m. 5 fur. 30 rd., and the other south 82 m. 6 fur. 16 rd.; how far were they then apart?

Ans. 148 m. 4 fur. 6 rd.

23. A farmer raised in one field 103 bu. 3 pk. 2 qt. of corn; in a second 205 bu. 6 qt., and in a third 310 bu. 3 pk.; how much did he raise in all?

Ans. 619 bu. 3 pk.

24. A ship sailed 15° 50′ 50″ at one time, and 31° 19′ 10″, in the same direction, at another; how many degrees was it then from the place of departure?

Ans. 47° 10′.

SUBTRACTION.

190. 1. Let it be required to find the difference between 19 cwt. 2 qr. 17 lb., and 15 cwt. 3 qr. 15 lb.

OPERATION.

	cwt.	qr.	lb.
	19	2	17
	15	3	15
Ans.	3	3	2

For convenience, we write the subtrahend under the minuend, so that denominations of the same kind stand in the same column, and begin at the right to subtract.

15 lb. from 17 lb. leaves 2 lb., which we write.

We cannot take 3 qr. from 2 qr., but we can take 1 cwt. from the 19 cwt., leaving 18 cwt., and the 1 cwt. taken is 4 qr., which added

How are the numbers written?

to the 2 qr. makes 6 qr.; 3 qr. from 6 qr. leaves 3 qr., which we write.

15 cwt. from 18 cwt. leaves 3 cwt., which we write.

Therefore, the difference required is 3 cwt. 3 qr. 2 lb.

Instead of making the upper number, 19 cwt., less by 1, the result would have been the same, if we had increased the corresponding lower number, 15 cwt., by 1.

Rule. — *Write the less number under the greater, so that denominations of the same kind shall stand in the same column.*

Beginning at the right, subtract each denomination of the subtrahend from the corresponding denomination of the minuend.

If the number of any denomination in the subtrahend is greater than that of the same denomination in the minuend, increase the upper number by as many units of that denomination as make one of the next higher, before subtracting; and consider the number of the next higher denomination of the minuend diminished by one.

PROOF. — The same as in simple numbers.

Examples.

	(2.)				(3.)			
	T.	cwt.	qr.	lb.	cwt.	qr.	lb.	oz.
From	14	17	2	16	3	0	0	14
Take	11	19	3	14	1	3	20	15
	2	17	3	2	1	0	4	15

REVIEW. — For what is Square Measure used? (159.) Solid Measure? (163.) Liquid Measure? (164.) Dry Measure? (165.) What is Time? (166.) For what is Circular Measure used? (175.)

How do you subtract? How do you proceed when any number of the minuend exceeds the number of the same denomination in the subtrahend? What is the Rule? The Proof?

(4.)

	oz.	pwt.	gr.
From	9	16	11
Take	4	19	21
	4	16	14

(5.)

	lb.	oz.	pwt.
	4	11	6
	2	10	11

(6.)

	yd.	ft.	in.
	4	1	9
	3	2	11
		1	10

(7.)

	fur.	rd.	yd.	ft.	
From	7	0	3	2	
Take	4	30	4	1	
	2	9	$4\frac{1}{2}$	1	in.
			$\frac{1}{2}=1$	6	
	2	9	4	2	6

(8.)

	R.	P.	sq. yd.	sq. ft.	sq. in.
From	3	25	6	8	116
Take	2	24	10	4	100
	1	0	$26\frac{1}{4}$	4	16
			$\frac{1}{4}=2$		36
	1	0	26	6	52

(9.)

	hhd.	gal.	qt.	pt.
From	23	50	2	1
Take	12	51	1	2
	10	62	0	1

(10.)

	hhd.	gal.	qt.	pt.
	19	44	2	0
	6	21	3	1

(11.)

	bu.	pk.	qt.
From	37	2	0
Take	19	3	7
	17	2	1

(12.)

	bu.	pk.	qt.
	54	0	0
	16	3	5

(13.)

	ch.	bu.
	31	19
	14	26

(14.)

	c. y.	d.	h.	m.
From	3	300	23	0
Take	1	350	0	40
	1	*315*	22	20

(15.)

	c. y.	d.	h.	m.	s.
	16	250	20	45	16
	14	300	22	15	17

16. From a chest containing 30 lb. 10 oz. 11 dr. of tea, have been taken 9 lb. 15 oz. 15 dr.; what quantity remains? Ans. 20 lb. 10 oz. 12 dr.

17. From a piece of calico containing 32 yd. 2 qr. 2 na., there have been cut off for a dress 11 yd. 3 qr. 2 na.; how much remains? Ans. 20 yd. 3 qr.

18. From a pile of wood containing 19 C. 86 cu. ft., there have been sold 13 C. 96 cu. ft.; how much remains?

Ans. 5 C. 118 cu. ft.

19. A farmer raised 106 bushels of wheat, and sold 36 bu. 3 pk. 2 qt. 1 pt.; how much had he left?

Ans. 69 bu. 5 qt. 1 pt.

20. If it were possible for a ship to sail around the world by a direct course, how much of its 360° of circumference would remain to be completed when 196° 55′ 42″ had been accomplished? Ans. 163° 4′ 18″.

191. The rule applies to finding the difference between two dates.

21. A man was born May 16, 1819; how old was he April 9, 1865?

OPERATION.

	y.	mo.	d.
	1865	4	9
	1819	5	16
Ans.	45	10	24

We write the *number* of the year, months, and days of the earlier date under that of the year, months, and days of the later date, and *count as many days to a month as are in the month next earlier than that named in the later date.*

Here that month has 31 days.

It is quite common to count every month as containing 30 days, which, in this case, would have given the difference 1 day less, and to that extent wanting in exactness.

REVIEW. — What is a number? (11.) A Simple Number? (187.) A Compound Number? (188.)

Explain the method of finding the difference between two dates.

22. A note was given April 30, 1865, and paid March 17, 1866; how long did it run?

23. Required the time between the discovery of America by Columbus, Oct. 12, 1492, and the Declaration of Independence, July 4, 1776.

MULTIPLICATION.

192. 1. Let it be required to find the product of 4 bu. 3 pk. 4 qt. 1 pt. multiplied by 7.

OPERATION.

	bu.	pk.	qt.	pt.
	4	3	4	1
				7
Ans.	34	0	7	1

For convenience, we begin with the lowest denomination to multiply. 7 times 1 pt. are 7 pt., which equal 3 qt. 1 pt.; we write the 1 pt., and reserve the 3 qt. to add to the product of the quarts.

7 times 4 qt. are 28 qt. which, with the 3 qt. added, are 31 qt., or 3 pk. 7 qt.; we write the 7 qt., and reserve the 3 pk. to add to the product of the pecks.

7 times 3 pk. are 21 pk., which, with the 3 pk. added, are 24 pk., or 6 bu. 0 pk.; we write the 0 pk., and reserve the 6 bu. to add to the product of bushels.

7 times 4 bu. are 28 bu., which, with the 6 bu. added, are 34 bu., which we write.

Therefore the product required is 34 bu. 0 pk. 7 qt. 1 pt.

Rule. — *Beginning at the right, multiply each denomination of the compound number in its order, and reduce each product to the next higher denomination; write the remainder, if any, underneath, and add the quotient to the next product.*

PROOF. — The same as in simple numbers.

With which denomination do you begin to multiply? How do you *reduce each product?* °Recite the Rule.

Examples.

(2.)

cwt.	qr.	lb.	oz.
4	2	16	10
			4
18	2	16	8

(3.)

T.	cwt.	qr.	lb.
17	16	3	10
			6

(4.)

lb.	oz.	pwt.	qr.
	4	10	5
			4

(5.)

lb.	oz.	pwt.	qr.
14	9	14	17
			5

(6.)

yd.	ft.	in.
4	1	2
		12
52	2	0

(7.)

m.	fur.	rd.
2	7	30
		6

(8.)

yd.	qr.	16ths.
5	3	2
		17
98	1	2

9. How many acres in an estate consisting of 5 farms, each measuring 70 A. 2 R. 14 P.?

Ans. 352 A. 3 R. 30 P.

10. How much wood in 4 ranges, each containing 5 cords, 6 cord feet? Ans. 23 C.

11. How much wine in 6 casks, each containing 49 gal. 2 qt. 1 pt.? Ans. 297 gal. 3 qt.

12. How much grain in 15 barrels, each containing 3 bu. 1 pk. 4 qt.?

13. Multiply 64 wk. 17 h. 38 m. by 11.

REVIEW. — How is Addition of Compound Numbers proved? Subtraction of Compound Numbers?

14. Required the weight of 3 dozen of silver spoons, if each dozen weigh 2 lb. 6 oz. 12 pwt. 3 gr.

Ans. 7 lb. 7 oz. 16 pwt. 9 gr.

15. If a planet move 2° 58′ 10″ per day, how far will it move in 56 days? Ans. 166° 17′ 20″.

16. If you should sleep 8 h. 15 m. 30 s. daily, how much will you sleep in 7 days?

17. What is the weight of 6 chests of tea, each weighing 2 qr. 16 lb. 8 oz.? Ans. 3 cwt. 3 qr. 24 lb.

18. How much land in 9 equal wood-lots of 113 A. 3 R. 25 P. each? Ans. 1025 A. 0 R. 25 P.

19. If you have for labor and recreation 16 h. 47 m. 30 s. each day, how many full days have you for that purpose in 8 days? Ans. 5 d. 14 h. 20 m.

DIVISION.

193. 1. Let it be required to find the quotient of 34 bu. 0 pk. 7 qt. 1 pt. divided by 7.

OPERATION.

bu.	pk.	qt.	pt.
7)34	0	7	1
4	3	4	1

For convenience, we begin with the highest denomination to divide. One seventh of 34 bu. is 4 bu., with a remainder of 6 bu., equal to 24 pecks; we write the 4 bu., and add the 24 pecks to the 0 peck in the dividend. 24 pk. and 0 pk. are 24 pk., and one-seventh of 24 pk. is 3 pk., with a remainder of 3 pk., equal to 24 quarts; we write the 3 pk., and add the 24 quarts to the 7 quarts in the dividend. 24 qt. and 7 qt. are 31 qt., and one-seventh of 31 qt. is 4 qt. with a remainder of 3 qt., equal to 6 pints; we write the 4 qt., and add the 6 pints to the 1 pint of the dividend. 6 pt. and 1 pt. are 7 pt., and one-seventh of 7 pt. is 1 pt., which we write.

Where do you begin to divide?

Rule. — *Beginning at the left, divide each denomination of the dividend, in its order; write the quotient, and reduce the remainder, if any, to the next lower denomination, adding the same to that denomination in the dividend, before dividing it.*

PROOF. — The same as in simple numbers.

When the divisor and dividend are both compound, or not of the same denomination, they must be reduced to the same denomination; and then the division is that of simple numbers.

Examples.

(2.)

cwt.	qr.	lb.	oz.
4)18	2	16	8
4	2	16	10

(3.)

	T.	cwt.	qr.	lb.
7)	2	15	1	21

(4.)

lb.	oz.	pwt.	gr.
4)1	6	0	20
	4	10	5

(5.)

	lb.	oz.	pwt.	gr.
9)	16	3	11	20
	1	9	14	$15\frac{5}{9}$

(6.)

M.	fur.	rd.
8)41	6	16
5	1	32

(7.)

	yd.	ft.	in.
2)	52	2	0

(8.)

	fur.	rd.	yd.
11)	35	10	1
	3	8	$1\frac{1}{11}$

9. I have 352 A. 3 R. 30 P., which I wish to divide into 5 equal farms; how many acres will there be for each?

10. Divide 69 sq. ft. 36 sq. in. by 9.

Ans. 7 sq. ft. 100 sq. in.

°Recite the Rule. What must be done when the divisor and dividend are both compound, or not of the same denomination?

11. Divide 6 T. 11 cwt. 3 qr. 16 lb. by 8.

Ans. 16 cwt. 1 qr. 23½ lb.

12. If 9 casks contain 412 gal. 3 qt. 1 pt., how much is that to a cask? Ans. 45 gal. 3 qt. 1 pt.

13. If 3 dozen spoons weigh 7 lb. 7 oz. 16 pwt. 9 gr., what is the weight of each dozen?

Ans. 2 lb. 6 oz. 12 pwt. 3 gr.

14. If 23 cords of wood be put into 4 equal ranges, how much will be in each range?

194. When the divisor is large, it is often convenient to write out the operation, after the manner of long division.

15. Divide 178 bu. 1 pk. by 23.

OPERATION.

23)178 bu. 1 pk. (7 bu.
161
17
4
68
1
23)69(3 pk.
69

Ans. 7 bu. 3 pk.

16. Divide 286 m. 6 fur. by 31.

OPERATION.

31)286 m. 6 fur. (9 m.
279
7
8
56
6
31)62(2 fur.
62

Ans. 9 m. 2 fur.

17. If a boarding-house requires 175 gal. 2 qt. of molasses in 52 weeks, how much is required a week?

Ans. 3 gal. 1 qt. 1 pt.

When the divisor is large, how is it often convenient to proceed?

18. Divide 675 yd. 3 qr. by 17. Ans. 39 yd. 3 qr.

19. If 15 loads of hay weigh 16 T. 10 cwt., what is the weight of 1 load? Ans. 1 T. 2 cwt.

20. If the moon moves in 29 days 382° 6′ 55″, how much is the daily motion? Ans. 13° 10′ 35″.

REVIEW.

1. Reduce 15 cwt. 3 qr. 11 lb. to pounds.

2. Reduce 13120 rods to miles. Ans. 41 miles.

3. Required the cost of 5 tons of iron, at 6 cents a pound. Ans. $600.

4. How many hundred-weight of flour can be bought, at 4 cents a pound, for $7.48? Ans. 1 cwt. 3 qr. 12 lb.

5. If a man travels one day 30 m. 2 fur. 23 rd., and another day 27 m. 3 fur. 17 rd., how far does he travel in all? Ans. 57 m. 6 fur.

6. If a merchant cut 11 yd. 3 qr. from a piece of calico containing 30 yd. 1 qr. 2 na., how much will remain?

7. If, from a cask containing 64 gal. 2 qt. 1 pt. 2 gi. of molasses, there should leak out 11 gal. 3 qt. 1 pt. 1 gi., how much would remain in the cask?

Ans. 52 gal. 3 qt. 0 pt. 1 gi.

8. What is the weight of 3 dozen silver forks, each fork weighing 4 oz. 0 pwt. 6 gr.?

Ans. 12 lb. 0 oz. 9 pwt.

9. How much must be paid for 2 ch. 28 bu. of coal, at 25 cents a bushel? Ans. $25.

10. Bought 3 pieces of cloth, each containing 42 yd., of which were sold one piece, and 27 yd. 1½ qr. of another; what quantity remained?

REVIEW. — What is Reduction? (180.) Reduction Descending? (181.) Reduction Ascending? (182.)

11. How much older is John than James, John's birth having been on the 20th December, 1858, and James' on the 10th of August, 1863? Ans. 4 yr. 7 mo. 21 d.

12. If a man can lift 201 lb. 12 oz., and that is 8 times what a boy can lift, how much can a boy lift?

Ans. 1 qr. 3 oz. 8 dr.

13. How many square yards of carpeting will it take for a room 26 ft. by 32 ft.? Ans. 92 sq. yd. 4 sq. ft.

14. If a steamer can cross the Atlantic, the distance being 3174 miles, in 15 days, what is her daily speed?

Ans. 211 m. 4 fur. 32 rd.

15. A milkman left 2 gal. 3 qt. 1 pt. of milk at a boarding house, every morning for 6 days; what quantity did he leave in that time? Ans. 17 gal. 1 qt.

16. Into how many house-lots, of 76 square rods each, can a field of 29 acres 40 square rods be divided?

Ans. 61, and 44 square rods remaining.

17 How much must be paid for a range of wood 100 feet long, 4 feet wide and 4 feet high, at $7.50 per cord?

18. How many days from March 4, 1871, to April 1, 1873? Ans. 759.

19. A man traveled one day, 15000 rods, another day, 31½ miles, and the third day, 25 miles, 80 rods. How many miles in all did he travel? Ans. 103⅝.

20. One quarter of an ox weighed 2 cwt. 2 qr. 6 lb., and the other three quarters weighed 3¼ times as much. What was the entire weight?

21. Multiply 24 d. 8 h. 42 m. 40 s. by 15, using its factors. Ans. 365 d. 10 h. 40 m.

22. Into how many lots of 3 A. 90 sq. rd. each, can a a farm of 174 A. 100 sq. rd. be divided?

Ans. 49, and 10 square rods remaining.

PERCENTAGE.

195. 1. How much is one hundredth of 100? Of 200?

2. How much is two hundredths of 100? Three hundredths of 100?

3. How many hundredths of 100 is 1? Are 2?

4. How many hundredths of 1, is $\frac{1}{10}$? Is $\frac{1}{2}$? Is 1?

5. How many hundredths of 1 is expressed by .03? By .06? By .25?

196. **Percentage** is the process of computing in hundredths.

The term *per cent.*, from *per*, by or on, and *centum*, hundred, means *by* or *on a hundred;* and 1 per cent. is 1 hundredth; 2 per cent., 2 hundredths; etc.

The **Rate** per cent. is the number of hundredths denoted by the per cent.

Any rate per cent. may be expressed in the form of a *decimal* or of a *common fraction*. Thus,

1 per cent.	is written	.01, or	$\frac{1}{100}$
5 per cent.	"	.05, "	$\frac{5}{100}$
10 per cent.	"	.10, "	$\frac{10}{100}$
25 per cent.	"	.25, "	$\frac{25}{100}$
100 per cent.	"	1.00, "	$\frac{100}{100}$
125 per cent.	"	1.25, "	$\frac{125}{100}$

$\frac{1}{2}$ per cent. is .00$\frac{1}{2}$, or .005; $\frac{1}{4}$ per cent. is .00$\frac{1}{4}$, or .0025; $\frac{1}{8}$ per cent. is .00$\frac{1}{8}$, or .00125, etc.

The **Sign** % is used for the word per cent.

What is Percentage? What is the term per cent. from? What does it mean? How may any rate per cent. be expressed?

The **Percentage** of a number is such a part of it as is denoted by the per cent.

The **Base** of percentage is the number upon which the percentage is computed.

EXERCISES.

Write decimally: —

1. 4 %.	4. $5\frac{1}{2}$ %.	7. 107 %.
Ans. .04.	Ans. .055.	Ans. 1.07.
2. 6 %.	5. $2\frac{1}{4}$ %.	8. 160 %.
Ans. .06.	Ans. .0225.	Ans. 1.60.
3. 9 %.	6. $\frac{1}{10}$ %.	9. 115 %.

Write in form of a common fraction in its lowest terms: —

10. 8 %.	13. $37\frac{1}{2}$ %.	16. 110 %.
Ans. $\frac{2}{25}$.	Ans. $\frac{3}{8}$.	Ans. $\frac{11}{10}$.
11. 20 %.	14. 75 %.	17. 150 %.
Ans. $\frac{1}{5}$.	Ans. $\frac{3}{4}$.	Ans. $\frac{3}{2}$.
12. 50 %.	15. 90 %.	18. 125 %.

MENTAL EXERCISES.

1. What is 5 per cent. of 10 bushels?

SOLUTION. — *Since* 1 *per cent. of* 10 *bushels is* 1 *hundredth of* 10 *bushels,* 5 *per cent. will be* 5 *hundredths of* 10 *bushels, which is* 50 *hundredths of* 1 *bushel, or* $\frac{1}{2}$ *of a bushel. Therefore,* 5 *per cent. of* 10 *bushels is* $\frac{1}{2}$ *of a bushel.*

2. How much is 9 per cent. of 10 yards?

3. How much is 6 per cent. of 12 dollars?

REVIEW. — What is a Decimal Fraction? (172.) What is the *denominator of a* Decimal Fraction? (174.)

4. If you earn \$24, and spend 4 per cent. of it, how much will you have left?

5. If I receive \$4 profit on \$100 of investment, how much is that on \$7?

SOLUTION. — *If the profit on* \$100 *is* \$4, *the profit on* 1 *dollar will be* 1 *hundredth of* \$4, *and on* \$7 *it will be* 7 *times* 1 *hundredth of* \$4, *which is* $\$\frac{28}{100}$, *or* \$.28. *Therefore, if the profit on* \$100 *is* \$4, *on* \$7 *it is* \$.28.

6. If I lose 11 yards of cloth on 100 yards, how much do I lose on 10 yards?

7. When I gain \$6 on \$100, how much do I gain on \$30?

8. Bought some articles for \$100, and sold them for \$112; how much do I gain on \$5?

9. What per cent. of 25 is 5?

SOLUTION. — *Since* 25 *is* 100 *per cent. of itself,* 1 *is* $\frac{1}{25}$ *of* 100 *per cent., and* 5 *is* 5 *times* $\frac{1}{25}$ *of* 100 *per cent., or* $\frac{5}{25}$ *of* 100 *per cent., which is* $\frac{1}{5}$ *of* 100 *per cent., or* 20 *per cent. Therefore,* 5 *is* 20 *per cent. of* 25.

10. What per cent. of 50 tons is 20 tons? Of 50 tons is 30 tons?

11. What per cent. of \$35 is \$7? Of \$40 is \$30?

12. If the cost of raising corn is 9 bushels to every 27 grown, what is the cost per cent.?

13. What per cent. of 28 bushels is 7 bushels?

14. If a bank receive \$7 for every \$100 lent, what per cent. does it receive?

15. If you should spend \$6 for every \$50 you earn, what per cent. of your earnings would you spend?

16. Bought a watch for \$100, and sold it for \$120; what per cent. did I make?

WRITTEN EXERCISES.

Case I.

To find any Per Cent. of a Number.

197. 1. What is 3 per cent. of $220?

OPERATION.

$220
.03
Ans. $6.60

Since 1 per cent. of $220 is 1 hundredth of $220, 3 per cent. will be 3 hundredths of $220, which is $220 × .03, or $6.60. Therefore, 3 per cent. of $220 is $6.60.

Rule.—*Multiply the given number by the rate per cent.*

Examples.

What is

2. 5 % of $72? Ans. $3.60.	6. $3\frac{1}{2}$ of 164 yd.? Ans. 5.74 yd.
3. 8 % of $90? Ans. $7.20.	7. $5\frac{1}{4}$ % of 8.75 bu.? Ans. .459+ bu.
4. 6 % of $112?	8. 7 % of 330 lb.?
5. 9 % of $38?	9. 20 % of 416 cu. ft.?

10. A farmer raised 160 bushels of apples, and sold 80 per cent. of the whole; how many did he sell?

11. Of $500, I have invested in the United States stocks 40 per cent., and the balance in land; how much have I invested in stock, and how much in land?

Ans. Stock $200; land $300.

12. From a deposit of $1800 in a bank, I have taken out 62 per cent.; how much remains? Ans. $684.

Explain the operation. What is the Rule?

Case II.

To find the Per Cent. one Number is of Another.

198. 1. What per cent. of 16 yards is 4 yards?

OPERATION.

$$\frac{4 \times 100}{16} = 25, \textit{Ans.}$$

Since 16 is 100 per cent. of itself, 1 is $\frac{1}{16}$ of 100 per cent., and 4 is 4 times $\frac{1}{16}$, or $\frac{4}{16}$, of 100 per cent., which is $\frac{1}{4}$ of 100 per cent., or 25 per cent. Therefore, 4 yards is 25 per cent. of 16 yards.

Rule.—*Multiply the number denoting the percentage by 100, and divide by the number on which the percentage is reckoned.*

Examples.

What per cent. is

2. 6 of 50? Ans. 12 per cent.
3. 7 of 140?
4. 9 of 150?
5. 15 of 90?
6. 7 lb. of 56 lb.? Ans. 12½ per cent.
7. $12 of $60?
8. 25 men of 225 men?
9. 21 miles of 280 miles?

10. If American gold coin is 9 parts pure gold to 1 part alloy, what per cent. is alloy? Ans. 10.

11. If a miller takes 4 quarts toll for grinding 32 quarts, what per cent. is the toll? Ans. 12½.

12. A farmer's flock of 120 sheep was increased to 156; what was the increase per cent.? Ans. 30.

13. I have a farm of 160 acres; 120 acres of it is under cultivation; what part is that of the whole? Ans. 75.

14. John is 12 years old, and his father 54 years; what per cent. is John's age of that of his father? Ans. 22⅔.

Explain the operation Recite the Rule.

15. I had 160 tons of coal, and sold 40 tons; what per cent. of what I had did I sell? Ans. 25.

COMMISSION.

199. A **Commission Merchant**, or **Factor**, is a person who buys or sells goods for another.

200. **Commission** is the percentage paid a commission merchant, or agent, for the transaction of business.

Commission is reckoned at a certain rate per **cent upon the sum expended or received,**

201. 1. A commission merchant sells coal to the amount of $550; what is his commission at 2 per cent.?

OPERATION.

$550
.02
Ans. $11.00

Since the commission on $1 is .02 of a dollar, on $550 it will be 2 hundredths of $550, or $550 × .02, which is $11. Therefore, his commission on $550, at 2 per cent., is $11.

Rule.—*Multiply the given sum by the number of per cent., expressed decimally.*

Examples.

2. How much must be paid for purchasing goods to the amount of $350, at 3 per cent. commission?

3. An agent sells goods to the amount of $1244; what is his commission at 2½ per cent.?

4. An auctioneer sold property to the amount of $1780; what were his fees at 1¼ per cent.?

What is a Commission Merchant, or Factor? What is Commission? Explain the operation. Recite the Rule.

PROFIT AND LOSS.

202. Profit and Loss is the process of estimating the gain or loss on capital employed in business.

Case I.

To find the Amount of Profit or Loss at any Per Cent.

203. 1. If a merchant buys goods to the amount of $1248, and sells them at a profit of 20 per cent., how much does he gain?

OPERATION.

$$\begin{array}{r} \$1248 \\ .20 \\ \hline \text{Ans. } \$249.60 \end{array}$$

Since the profit is 20 per cent. of $1248, it must be 20 hundredths of $1248, or $1248 × .20, which is $249.60. Therefore, the gain on $1248, at 20 per cent. profit, is $249.60.

Rule. — *Multiply the number upon which the percentage is reckoned, by the number of per cent., expressed decimally.*

What is Profit and Loss? Recite the Rule.

Examples.

2. A cistern containing 120 gallons lost by leakage 5 per cent. of its contents; how much was the loss?

Ans. 6 gallons.

3. If United States stocks bought for \$300 should advance 9½ per cent., what would be the gain?

Ans. \$28.50.

4. Bought a horse for \$216, and sold him at a loss of 10 per cent.; how much was the loss? Ans. \$21.60.

Case II.

To find the Rate Per Cent. of Gain or Loss.

204. 1. If I buy a barrel of flour for \$8, and sell it for \$10, what is the gain or loss per cent.?

OPERATION.

\$10 — \$8 = \$2; $\frac{\$2 \times 100}{\$8} = 25$, *Ans.*

\$10 — \$8 = \$2, the sum gained. Then, since 8 is 100 per cent. of itself, 1 is ⅛ of 100 per cent., and 2 is ⅛ of 100 per cent., which is ¼ of 100 per cent., or 25 per cent. Therefore, if I buy a barrel of flour for \$8 and sell it for \$10, I gain 25 per cent.

Rule. — *Multiply the number denoting the gain or loss by 100, and divide by the number on which the gain or loss is reckoned.*

Examples.

2. Bought a cord of wood for \$6, and gained in selling it \$.72, what was the gain per cent.?

Explain the operation. Recite the Rule.

3. When molasses is bought for 90 cents a gallon, and sold at 108 cents, what is the gain per cent.?

4. Bought a horse for $125, and sold it for $12.60 less than cost; what per cent. was the loss?

5. If I sell a farm which cost me $5000 for $4700, what is the loss per cent.? Ans. 6 per cent.

INTEREST.

205. Interest is percentage allowed for the use of money, or for value received.

The **Principal** is the sum on interest.

The **Rate** of interest is the per cent. allowed on the principal for one year, or any given time.

The **Amount** is the sum of the principal and interest.

206. A Legal Rate is the rate per cent. established or allowed by law. The legal rate in most of the States, and in the United States Courts, when no particular rate is mentioned, is 6 per cent.

In New York, New Jersey, Michigan, Wisconsin, Minnesota, South Carolina, and Georgia it is 7 per cent.; in Louisiana it is 5 per cent.; in a few States 8 per cent.; and in some States as high as 10 per cent.

The rate for one year, and at 6 per cent., is to be understood in this book when no other is named.

207. Since the interest of $1, at 6 per cent. for 1 year, or 12 months, is 6 cents, for 1 month it will be $\frac{1}{12}$ of 6

What is Interest? The Principal? The Rate? The Amount? Legal Rate? What rate is to be understood when no other is named?

cents, which is 5 mills, and for two months, 2 times 5 mills, or 1 cent; also,

Since the interest for 1 month, or 30 days, is 5 mills, the interest for 6 days, or $\frac{1}{5}$ of 30 days, will be 1 mill, and for 2 days, 3 days, etc., as many sixths of a mill as there are days. Hence, the

Table.

Interest of $1, at 6 per cent.,

For	12	months,	or	1	year,	is $0.06
"	2	months,	"	$\frac{1}{6}$	year,	" 0.01
"	1	month,	"	$\frac{1}{12}$	year,	" 0.005
"	6	days,	"	$\frac{1}{5}$	month,	" 0.001
"	1	day,	"	$\frac{1}{30}$	month,	" $0.000\frac{1}{6}$.

Case I.

The Interest of any Sum at 6 Per Cent.

208. 1. What is the interest of $212 for 2 y. 7 mo. 15 da.?

OPERATION.

Interest of $1,
For 2 years = $.12
" 7 months = .035
" 15 days = $.002\frac{1}{2}$
= $.157\frac{1}{2}$

Principal, $212
$.157\frac{1}{2}$
1484
1060
212
106
Ans. $33.390

Recite the Table.

Since the interest of $1 for one year is 6 cents, for 2 years it is 12 cents, for 7 months ½ of 7 cents, or 3 cents 5 mills, and for 15 days ½ of 15 mills, or 2½ mills; and taking the sum, we have the interest for 2 y. 7 m. 15 da., $.157½. If the interest of $1 is $.157½, the interest of $212 is 212 times $.157½, which is $33.39. Therefore, the interest required is $33.39.

Rule.—*Find the interest of $1 for the given time, by reckoning for* **years** *six times as many cents as there are years; for* **months** *one half as many cents as there are months; and for* **days** *one sixth as many mills as there are days; and multiply the principal by the number denoting that interest.*

Examples.

What is the interest

2. Of $187.25 for 1 year 4 months?
Ans. $14.98.
3. Of $98.20 for 1 year 28 days?
4. Of $126.46 for 9 months? Ans. $5.69.
5. Of $194 for 4 months 12 days? Ans. $4.268.
6. Of $318 for 10 months 16 days?
7. Of $320 for 33 days? Ans. $1.76.
8. Of $500 for 93 days? Ans. $7.75.
9. Of $620 for 2 years 3 months 3 days?
10. Of $715 for 1 year 5 months 10 days?
Ans. $61.976.
11. Of $812 for 3 years 6 months 6 days?
Ans. $171.332.
12. Of $750 for 2 years 9 months 18 days?

Explain the operation. Recite the Rule.

Case II.

The Interest of any Sum at any Rate.

209. 1. Required the interest of $640 for 3 years 7 months 3 days, at 7 per cent.

FIRST OPERATION,		SECOND OPERATION.
$640		$640
.215½		.07
3200	Int. 1 yr.,	$44.80
640		3
1280		
320	Int. 3 yr.,	$134.40
	Int. ½ yr., or 6 mo.,	22.40
6)$137.920	Int. ⅙ of 6 mo., or 1 mo.,	3.733+
22.986+	Int. 1/10 mo., or 3 da.,	.373+
Ans. $160.906+		Ans. $160.906+

Since the interest of $1 for the given time, at 6 per cent., is $.215½, the interest of $640 is 640 times $.215½, which is $137.92; and one sixth of this, or $22.986+, is the interest at 1 per cent., which, added to the interest at 6 per cent., gives $160.906+, as the interest at 7 per cent.

Again, since the interest of $1 for 1 year, is $.07, the interest of $640 for 1 year is 640 times $.07, which is $44.80, and for 3 years is 3 times $44.80, which is $134.40; 7 months equal 6 months plus 1 month; and since 6 months equal ½ of a year, the interest for 6 months is ½ of $44.80, which is $22.40; and since 1 month is ⅙ of 6 months, the interest is ⅙ of $22.40, which is $3.733+; and since 3 days are 1/10 of a month, the interest for 3 days is 1/10 of $3.733+, which is $.373+; and, taking the sum of these, we have as the whole interest $160.906+

Explain the first operation. The second.

Therefore, the interest of $640 for 3 years, 7 months and 3 days, at 7 per cent. is $160.906+.

Rule. — *Find the interest for* **years** *by multiplying the principal by the rate expressed decimally, and that product by their number; for* **months,** *take such a part of the interest for one year as their number is part of a year; and for* **days,** *take such a part of the interest of one month as their number is part of a month. Or,*

Find the interest at 6 per cent., and increase or diminish as the per cent. is greater or less than 6 per cent.

For 7 per cent., add to 6 per cent. $\frac{1}{6}$; for 8 per cent., $\frac{1}{3}$; for 9 per cent $\frac{1}{2}$; for 5 per cent. subtract $\frac{1}{6}$; for 4 per cent., $\frac{1}{3}$, etc.

Examples.

2. What is the interest of $780 for 5 years, at 7 per cent.? Ans. $273.

3. What is the interest of $30.50 for 3 years, at 5 per cent.? Ans. $4.575.

4. What is the interest of $130.60 for 6 months, at 7 per cent.? Ans. $4.571.

5. What is the interest of $516 for 9 months, at 7 per cent.?

6. What is the interest of $234.92 for 33 days, at 7 per cent.? Ans. $1.507+.

7. What is the interest of $420 for 63 days, at 9 per cent.? Ans. $6.615+.

What is the Rule? For 7 per cent., what is added to 6 per cent.? For 8 per cent.? For 9 per cent.? For 5 per cent., what is subtracted? For 4 per cent?

8. What is the interest of $226 for 2 years and 3 months, at 5 per cent.? Ans. $25.425.

9. What is the interest of $144.20 for 3 years 8 months, at 10 per cent.? Ans. $52.873.

10. What is the interest of $210, for 4 years 5 months, at 7 per cent.?

11. What is the interest of $67.42, for 1 year 7 months 15 days, at 6 per cent.? Ans. $6.573+.

12. What is the interest of $333 for 5 years 2 months 5 days, at 8 per cent.? Ans. $138.01.

210. When the amount is required, add the principal and interest. (Art. 205.)

13. What is the amount of $263.48, for 2 months 21 days, at 6 per cent.? Ans. $267.036+.

14. What is the amount of $54.98 for 6 months 20 days, at 7 per cent.?

15. What is the amount of $45.15 for 1 year 4 months 9 days, at 5 per cent.? Ans. $48.216+.

16. What is the amount of $100 from May 16, 1865, to August 31, 1867, at 10 per cent.? Ans. $122.916+.

BANK DISCOUNT.

211. **Bank Discount** is interest paid in advance on a sum promised in a *note*.

212. A **Note** is not considered due till 3 days after the time mentioned in it. Hence,

---

REVIEW. — What is Percentage? (194.) What is Commission? (199.) Profit and Loss? (202.) Interest? (205.) The Principal? (205.) *The Amount?* (205.)

In computing bank discount, 3 days called **Days of Grace**, are added to the specified time.

FORM OF NOTE PAYABLE AT BANK.

$500. Trenton, April 29, 1871.

Ninety days after date, I promise to pay William H. Harlow, or order, at the Second National Bank, Five Hundred Dollars. Value received.

JAMES BLAIR.

EXERCISES.

213. 1. What is the bank discount on the above note at 6 %?

OPERATION.

Int. of $500 *for* 60 *d.* $= \frac{1}{100}$	*of* $500	$=	$5.00 *discount.*
" " " 30 *d.* $= \frac{1}{2}$ *of* $\frac{1}{100}$	"	$=	2.50 "
" " " 3 *d.* $= \frac{1}{10}$ *of* $\frac{1}{2}$ *of* $\frac{1}{100}$	"	$=	.25 "
Int. of $500 *for* 93 *d.*		$=	$7.75 "

Since the interest of $1 for 60 days is 1 cent, or .01, of the principal, the interest of $500 must be, for 60 days, $\frac{1}{100}$ of $500, or $5.00; for 30 days, or $\frac{1}{2}$ of 60 days, it must be $\frac{1}{2}$ of $\frac{1}{100}$ of the principal, or $2.50; and for 3 days, or $\frac{1}{10}$ of 30 days, it must be $\frac{1}{10}$ of $\frac{1}{2}$ of $\frac{1}{100}$ of the principal, or $.25. Hence, the interest for 93 days is $5.00 + $2.50 + $.25 = $7.75, which is the discount required.

2. What is the bank discount on a note for $600, on 60 days, at 7 %? Ans. $7.35.

3. What is the bank discount on a note for $375.50, on 120 days, at 8 %?

4. Taking out the bank discount on a note for $3000, on 30 days, at 6 %, how much of its face will remain?

Ans. $2983.50.

What is Bank Discount? What are called Days of Grace? Explain the operation.

STOCKS AND BROKERAGE.

214. **Stocks** is a general term applied to the bonds of governments, and to the bonds and shares of incorporated companies.

215. The principal bonds of the United States are:—

6's of 1881, extended, bearing interest at $3\frac{1}{2}$ %.
5's of 1881, extended, bearing interest at $3\frac{1}{2}$ %.
$4\frac{1}{2}$'s of 1886, bearing interest at $4\frac{1}{2}$ %.
4's of 1901, bearing interest at 4 %.

216. The **Par Value** of stocks is their face value.

217. The **Market Value** of stocks is quoted at a certain per cent. of the par value. Thus,

218. Bonds, or shares, quoted at 100, are *at par;* at less than 100 are *below par*, and at more than 100 are *above par.*

219. A **Broker** is a dealer in stocks; and Brokerage is his commission for buying or selling.

220. **Brokerage** is reckoned on the face value of stocks.

EXERCISES.

1. What is the market value of $5000 of U. S. 6's of 1881, at 112? $5600.

2. How much must be paid for $1500 of State bonds at 102, and brokerage at $\frac{1}{4}$ %? Ans. $1533.75.

What are Stocks? What is the Par Value of a Stock? The Market Value? What is a Broker? Brokerage?

3. What is the brokerage at $\frac{1}{4}$ %, for purchasing 17 shares of National Bank stock, the face value of each share being $100?

4. What rate per cent. of profit will be realized from a 10 per cent. stock bought at 120? Ans. $8\frac{1}{3}$.

5. How much will $6000 of 5's of 1881 cost at 106 and brokerage $\frac{1}{4}$ %? Ans. $6375.

GENERAL REVIEW.

1. A merchant receives from the sale of coffee $519.60, of tea $310.20, and of sugar $640.45; how much does he receive from the whole? Ans. $1470.25.

2. If you should buy a farm for $5600, and sell a part of it for $3780, and the remainder for $2340, how much would you make by the operation?

3. The difference of two numbers is 397, and the smaller number is 532; what is the larger number?

4. A and B together had $655, but A having lost $65, they then had equal sums; how much had B?

5. A merchant bought 55 bales of cloth, each bale containing 36 pieces, and each piece 30 yards; how many yards did he buy in all? Ans. 59400.

6. Two travelers set out from towns 363 miles distant, and travel towards each other till they meet, when it appears one has traveled 93 miles farther than the other; how far had each traveled?

REVIEW. — What is Quantity? (10.) Numbers? (11.) Arithmetic? (15.) Figures? (16.) Notation? (32.) Numeration? (36.) What are principles of Notation? (34.)

7. If the product of two numbers is 1960, and one of the numbers is 35, what is the other number?

8. If I have \$3642, and purchase 404 barrels of flour at \$9 a barrel, how much shall I have left after paying for the flour? Ans. \$6.

9. How many bushels of wheat, at \$1.25 a bushel, will pay for 10 yards of broadcloth, at \$4.50 per yard?

Ans. 36 bushels.

10. Divide the product of 35×20 by $640 - 515$.

11. If 9 acres of grass will pasture 15 cows, how many cows will 6 acres pasture?

12. When 141 tons of coal can be bought for \$775.50, how much is the cost of 100 tons?

13. Required the prime factors of 1365.

14. Reduce $312\frac{4}{5}$ to an equivalent improper fraction.

15. Reduce 1564 to an equivalent fractional form.

16. Change $\frac{3}{5}$ and $\frac{4}{7}$ to equivalent fractions having a common denominator. Ans. $\frac{21}{35}$ and $\frac{20}{35}$.

17. A merchant sold, at one time, $\frac{2}{5}$ of a ship, at another time, $\frac{1}{2}$ of the remainder; what part of the whole ship did he sell at both times? Ans. $\frac{7}{10}$.

18. $\frac{2}{5} + \frac{3}{10}$ subtracted from 1 leaves how many?

19. What is the cost of 6 acres and 2 roods of land, at \$12.50 a rood? Ans. \$325.

20. What is the difference of time between Jan. 30, 1865, and May 16, of the same year?

21. How many $1\frac{1}{2}$ pint bottles can be filled from a cask of wine containing 54 gal. 1 qt. 1 pt.?

REVIEW. — Name the orders in the Numeration Table. (33.) What is the Rule for Addition? (42.) For Subtraction? (47.) For Multiplication? (49.) For Short Division? (64.) For Long *Division? (66.)*

22. Gave $4\frac{5}{8}$ for a vest, $10\frac{3}{4}$ for a coat, and $5\frac{3}{16}$ for boots; how much did the whole cost?

23. At $\frac{3}{8}$ of a dollar a yard, how many yards of cloth can be had for $12? Ans. 32 yards.

24. What decimal is equivalent to $\frac{1}{1}\frac{3}{6}$?

25. What is the difference between thirty thousandths and thirty thousand? Ans. 29999.970.

26. Purchased two loads of hay, the first weighing 16 cwt. 3 qr. 14 lb., and the second 17 cwt. 2 qr. 11 lb.; what was the weight of the two loads?

27. If 105 is $\frac{3}{7}$ of some number, what is that number? Ans. 245.

28. When $5500 is the value of $\frac{1}{2}$ of $\frac{5}{8}$ of a cotton factory, what is the whole of it worth?

29. How long will 2295 pounds of beef last a garrison of 45 men, if each man should get 1 pound 3 times a week? Ans. 17 weeks.

30. A farmer, being asked how many sheep he had, answered, that he had them in 4 fields; in the first he had $\frac{1}{4}$ of his flock, in the second $\frac{1}{5}$, in the third $\frac{1}{6}$, and in the fourth 46; how many had he in all?

31. What common fraction is equivalent to .9375?

32. What per cent. of 60 is 15?

33. What is the quotient, when 500 is the dividend and .05 the divisor? Ans. 10000.

34. Required the quotient of .05 divided by 500.

35. What is the product of .0001 multiplied by 500? Ans. .05.

REVIEW. — What is United States Money? (69.) How may United States Money be written, added, etc.? (74.) What is Reduction? (181.) What is the Rule for Reduction Descending? (182.)

36. If the number of acres in a certain town is 12,935, and the total valuation of real estate is $714,200, what is the average value of real estate to the acre?

Ans. $55.21+.

37. What is an auctioneer's commission for selling goods to the amount of $16000, at 2¼ per cent.?

38. A farmer divided his real estate, consisting of 113 A. 3 R. 25 P., equally among his 9 children; how many acres did each receive? Ans. 12 A. 2 R. 25 P.

39. How much wood is contained in a load 8 feet long, 3½ feet wide, and 5½ feet high? Ans. 1 C. 26 cu. ft.

40. If ⅞ of a ship cost $35000, what is ½ of it worth?

Ans. $20000.

41. How many yards of cloth 1¼ yards wide will line 16 yards ⅝ of a yard wide? Ans. 4⅘ yards.

42. Bought a horse for $125, but not proving as good as expected, I sold him for $110; what was the loss per cent.? Ans. 12 per cent.

43. Bought a house for $3600, and sold it at a loss of 7 per cent.; how much did I obtain for it? Ans. $3348.

44. Required the interest on $27.60 for 6 years, 6 months, and 6 days, at 6 per cent. Ans. $10.791+.

45. If the interest of $350 for 1 year is $21, what is the interest of $250 for the same time?

46. What number multiplied by 12½ gives 1000?

47. What number divided by 7¾ gives 110?

48. If I buy corn for 50 cents a bushel, and sell it at an advance of 20 per cent., how many cents do I gain per bushel?

REVIEW. — Reduction Ascending? (185.) What is a simple Number? (187.) A Compound Number? (188.) A Factor? (87.) *Factoring?* (88.) Cancellation? (192.) Analysis? (94.)

49. The sum of two numbers is 93, and one of them is $11\frac{1}{5}$; what is the other?

50. If I buy corn for 60 cents a bushel, and sell it at a loss of 20 %, what do I get a bushel for it?

51. Of a certain pole, 5 feet stands in earth, 10 feet in water, and $\frac{2}{5}$ above water; what is the length of the pole? Ans. 25 feet.

52. What fraction is that to which if you add $\frac{1}{7}$ the sum will be $\frac{2}{3}\frac{7}{8}$? Ans. $\frac{1}{4}$.

53. How many rectangular blocks, $1\frac{1}{2}$ feet wide and 2 feet long, will pave a cellar floor 15 feet wide and 16 feet long? Ans. 80 blocks.

54. When the dividend is 17.28, and the quotient 14.4, what is the divisor? Ans. 1.2.

55. When the dividend is .001, and the quotient .000001, what is the divisor? Ans. 1000.

56. Bought a horse, and paid \$72 down, which was $\frac{6}{11}$ of the price; what was the price? Ans. \$132.

57. What is the interest on \$600 for 6 months, at $7\frac{3}{10}$ per cent.?

58. Bought a cask of molasses, containing 65 gallons, for \$26; for how much per gallon must I sell it to gain $12\frac{1}{2}$ per cent.? Ans. \$.45.

59. What is the value of a pile of wood 36 feet long, 4 feet wide, and 6 feet high, at \$6 per cord?

60. How many steps, of 2 feet 6 inches each, must a man take, in walking a mile? Ans. 2112 steps.

61. If the cost of $\frac{7}{8}$ of a farm be \$3500, what is $\frac{3}{4}$ of it worth? Ans. \$3000.

REVIEW. — What is a Fraction? (96.) A Common Fraction? (99.) A Decimal Fraction? (129.) From what is the term per cent. derived? (196.)

62. What is the value of a small rectangular farm, 72 rods long and 40 rods wide, at $50 per acre?

63. A frog, at the bottom of a well 15 feet deep, commenced going towards the top. In his journey he got up 3 feet every day, and fell back 2 feet every night; how many days did it take him to get out of the well?

64. What is the interest on a note of $500 from Jan. 28, to May 21, 1867, at 7 per cent.? Ans. $10.986+.

65. A man purchased a hat worth $5, and handed the merchant a $50 bill in payment; but the merchant, not being able to make change, passed it to a broker, and on getting it changed, he gave the purchaser of the hat his change. After the customer had left, the broker, finding the bill to be counterfeit, returned it to the merchant, and received good money for it. How much did the merchant lose by the transaction?

66. A farmer being asked how many sheep he had, said, "If you add to $\frac{1}{2}$ of the number, $\frac{1}{2}$ of $\frac{3}{4}$ of it, and 5 sheep, you will have the number;" how many had he?

67. The population of the United States in 1860 was 31,000,000, and if the increase had been at the rate of 30 per cent. for the next 10 years, what would have been the population in 1870? Ans. 40,300,000.

68. A man gave $\frac{1}{3}$ of his property to his wife; $\frac{1}{2}$ of the remainder to his 3 sons, and the balance of what he had to his 5 daughters. The daughters received each $600. What was the entire property? How much did the wife and each son receive? Ans. Whole property, $9000; wife received $3000; each son, $1000.

REVIEW. — What is Commission? (199.) Profit and Loss? (202.) *What* is Interest? (205.) What is the Rule for finding the interest *on any sum, for any rate*? (209.)

APPENDIX.

METRIC SYSTEM.

221. The **Metric System** is so called from the meter, which is the unit from which all its weights and measures are derived.

It was authorized to be used in the United States, in 1866.

222. The unit of any measure being named, the names of the *higher denomination* are formed by prefixing to the unit name

DEKA,	HEKTO.	KILO,	MYRIA,
10,	100,	1000,	10000;

and the names of the *lower denominations* by prefixing

DECI,	CENTI.	MILLI,
.1,	.01,	.001.

These prefixes are pronounced respectively, dĕk'a, hĕk'to, kĭl'o, mўr'ia, dĕc'i, cĕn'ti, and mĭl'li.

MEASURES OF LENGTH.

223. The **Meter**, the unit of length, is one ten-millionth of the distance on the earth's surface from the equator to the pole.

Table.

10 *millimeters*	(mm.)	are 1 *cen'timeter*,	cm.
10 centimeters,		1 dec'imeter.	
10 decimeters,		1 METER,	m.
10 meters,		1 dek'ameter.	
10 dekameters,		1 hek'tometer.	
10 hektometers,		1 KIL'OMETER,	Km.
10 kilometers,		1 myr'iameter.	

Why is the Metric System so called? How are the names of the higher denominations formed? Of the lower denominations? What is the Meter? Recite the Table.

The *meter* is the unit of measure for all common lengths and distances, and is equal to 39.37 inches, or 3.28 feet.

The *kilometer* is taken as the unit in measuring very long distances, — as the length of roads, distances between cities, etc. It is equal to .62137 of a mile, or 3280 feet 10 inches.

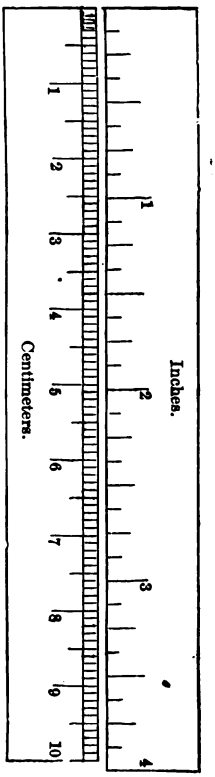
A *meter* is about 3 feet, 3 inches, and 3 eighths of an inch, and a *kilometer* is about 200 rods, or $\frac{5}{8}$ of a mile.

25 millimeters nearly replace the *inch*, 3 decimeters the *foot*, 5 meters the *rod*, and 1600 meters the *mile*.

The nickel 5-cent pieces are each 20 millimeters, or 2 centimeters, in diameter; so that 50 of these coins, side by side in a straight line, will measure a meter.

The scales in the margin exhibit, the one a decimeter or tenth of a meter, divided into centimeters and millimeters, and the other, four inches divided into eighths of an inch.

In the above Table, and in those that follow, the principal unit is designated by capitals, and other units, which are often used, by italics.



A DECIMETER.

MEASURES OF SURFACE.

224. The **Square Meter**, the principal unit for the measure of surface, is the square whose side is one meter.

Table.

100 sq. *millimeters* (sq. mm.)	are	1 sq. *centimeter*,	sq. cm.
100 sq. centimeters,		1 sq. decimeter.	
100 sq. decimeters,		1 sq. METER,	sq. m.

To what is the Meter equal? The Kilometer? About how much *is a Meter? What is the* Square Meter? Recite the Table.

The *square meter* is the unit for measuring all surfaces except land, and is equal to 1.196 square yards, or 10.76 square feet.

225. The **Are** (pronounced *air*), the unit of land measure, is the square whose side is ten meters.

Table.

100 cen'tares (ca.)	or sq. meters	are	1 are, a.
100 ares,			1 *hec'tare*, Ha.

An *are* is equal to 119.6 square yards, or 3.953 square rods, and a *hektare* to 2.471 acres.

A *centare* is about 1⅛ square yards, and a *hektare* is very nearly 2½ acres. 40 ares nearly replace an acre.

MEASURES OF VOLUME.

226. The **Cubic Meter**, the principal unit for the measure of volume, is the cube whose edge is one meter.

Table.

1000 *cu. millimeters* (cu. mm.)	are	1 *cu. centimeter*, cu. cm.
1000 cu. centimeters,		1 cu. decimeter.
1000 cu. decimeters,		1 CU. METER, cu. m.

A *cubic meter* is equal to 35.3166 cubic feet, or 1.308 cubic yards.

227. The **Liter** (pronounced *leeter*), the unit of capacities, is a cubic decimeter.

Table.

10 mil'liliters (ml.)	are	1 *cen'tiliter*, cl.
10 centiliters,		1 dec'iliter.
10 deciliters,		1 LITER l.
10 liters,		1 dek'aliter.
10 dekaliters,		1 HEK'TOLITER, Hl.
10 hektoliters,		1 kil'oliter.

The *liter* is used in measuring liquids, and is 61.022 cubic inches, or 1.0567 liquid quarts or .908 dry quarts.

To what is a Square Meter equal? What is the Are? Recite the Table. To what is an Are equal? Recite the Table. What is a Cubic Meter? Recite the Table. What is a Liter? Recite the Table. To what is a Liter equal?

The *hektoliter*, or for brevity *hekto*, is used in measuring dry articles, such as grains, salt, etc., and is 2.837 bushels, or 3.531 cubic feet.

A *liter* is about $1\frac{1}{18}$ liquid quart; a *hektoliter* about $2\frac{5}{8}$ bushels, or $\frac{5}{8}$ of a barrel; 4 liters a little more than replace the *liquid gallon*, and 35 liters very nearly the common *bushel*.

228. The **Stere** (pronounced *stair*), the measuring unit for wood, is a cubic meter, or 1000 liters.

Table.

10 deci'steres	are 1 STERE,	st.
10 steres,	1 dek'astere.	

A *stere* is equal to 1.308 cubic yards, or .2759 cord; 36 decisteres, or 3.6 steres, very nearly replace the cord.

WEIGHTS.

229. The **Gram**, the principal unit of weight, is the weight, in a vacuum, of a cubic centimeter of distilled water, at its greatest density.

Table.

10 *mil'ligrams* (mg.)	are 1 cen'tigram.	
10 centigrams,	1 dec'igram.	
10 decigrams,	1 GRAM,	g.
10 grams,	1 dek'agram.	
10 dekagrams,	1 hektogram.	
10 hektograms.	1 KIL'OGRAM,	Kg.
10 kilograms,	1 myr'iagram.	
10 myriagrams,	1 quin'tal.	
10 quintals,	1 METRIC TON.	T.

The *gram* is used in mixing medicines, weighing letters, jewels, etc. It is equal to 15.432 grains.

The *kilogram*, or for brevity *kilo*, is the ordinary weight of commerce. It is equal to 2.2046 pounds, or 35.27 ounces.

For what is the Hektoliter used? About how much is the Liter? The Hektoliter? What is the Stere? Recite the Table. To what is the Stere equal? How many Decisteres nearly replace the Cord? What is the Gram? Recite the Table. For what is the Gram *used? The Kilogram?*

The *metric ton* is used in weighing heavy articles, and is 2204.6 pounds, or 1.1023 common tons.

A *kilo* is about $2\frac{1}{5}$ pounds, and 28 *grams* nearly replace the avoirdupois ounce.

The nickel 5-cent pieces each weigh 5 grams.

NOTATION AND NUMERATION.

230. In writing by figures numbers expressing metric weights and measures, the decimal point is placed between the unit, and its subdivisions written as decimal orders.

One, two, or three orders of figures are allowed to each denomination lower than the unit, according as 10, 100, or 1000 of a lower denomination make one of the next higher. Thus,

3 meters, 4 decimeters, and 6 centimeters, is written as meters, 3.46 $^{m.}$

6 hektares, 5 ares, and 25 centares, as hektares, 6.0525 $^{Ha.}$

165 cubic meters and 17 cubic centimeters, as cubic meters, 165.000017 $^{m.}$

EXERCISES.

Write in figures: —

1. 8 metric tons, 6 myriagrams, and 4 kilograms, as metric tons.

We write the 8 metric tons as the integral part of a mixed number, and place a decimal point at the right. Then, since there are no quintals, we write 0 in the tenths place; we write the 6 myriagrams in the hundredths place; the 4 kilograms in the thousandths place; and have 8.064 $^{T.}$

2. Seventeen meters, six decimeters, and five centimeters.

Ans. 17.65 $^{m.}$

3. Eleven hektares, twenty-five ares, and five centares, as ares.

Ans. 1125.05 $^{a.}$

4. Fifty-six steres and three decisteres. Ans. 56.3 $^{st.}$

For what is the metric ton used? To what is a gram equal? A kilo? A metric ton? How are figures denoting the subdivisions of the unit written? How many figures are allowed to each denomination lower than the unit?

5. One hundred and fifty square meters, and seventy-five square centimeters. Ans. 150.0075 sq. m.

6. Thirteen hektoliters, seven liters, and five centiliters, as liters. Ans. 1307.05 l.

7. Eight hektograms, four decigrams, as grams.

8. Three cubic meters, seven hundred cubic decimeters, and fifteen cubic centimeters. Ans. 3.700015 m.

9. Three metric tons, three kilograms, and seven grams, as kilograms. Ans. 3003.007 Kg.

10. Fifteen hektares, seventy-three ares, and five centares, as hektares. Ans. 15.7305 Ha.

11. Twenty-five hektoliters, seven dekaliters, and four liters, as hektoliters. Ans. 25.74 Hl.

12. Five hundred seven metric tons, five quintals, seven myriagrams, and two kilograms, as metric tons. Ans. 507.572 T.

231. The *integer* of a metric expression is usually read as a number of its primary unit, and the decimal part, if any, as a number of the lowest denomination denoted. Thus,

3.46 m. may be read as three meters, and forty-six centimeters.

6.0525 Ha. as six hektares, and five hundred twenty-five centares.

165.000017 m. as one hundred sixty-five cubic meters, and seventeen cubic centimeters.

EXERCISES.

1. Copy and read 28.42 m.

This is a mixed number, expressing 28 meters, and 42 hundredths of a meter, and may be read twenty-eight meters and forty-two centimeters.

Copy and read: —

2.	148.165 Km.	6.	37.05 l.
3.	31.0005 m.	7.	95.7 st.
4.	47.0080 Ha.	8.	567.450 Kg
5.	650.675008 cu. m.	9.	647.08 Hl.

How is the integer of a metric expression usually read? The decimal part?

DICTATION EXERCISES.

232. **Dictation Exercises** are here supplied for the convenience of teachers, who may desire, for any purpose, more exercises than are to be found in the body of the book.

NOTATION AND NUMERATION.

Mental Exercises.

233. 1. How many units in five ?

2. How many units in nine dollars?

3. Are seven and eight like or unlike numbers ?

4. Are two horses and four trees like or unlike numbers ?

5. Name some like numbers.

6. How many ones in ten? In ten and five units ?

Written Exercises.

234. Write in figures the following numbers : —

1. Eighty-three.
2. Thirty-eight.
3. Ninety-nine.
4. Eight hundred three.
5. Nine hundred ninety.
6. One hundred ten.
7. One hundred one.
8. Four hundred seventeen.
9. Seven hundred seventy-four.
10. One thousand forty-one.
11. Seven thousand seven hundred twenty-seven.
12. Seven hundred seventy-seven thousand one hundred.

235. Point off and read the following: —

1.	163.	6.	310036.	11.	16181919.
2.	480.	7.	72935.	12.	257800808.
3.	7101.	8.	62625.	13.	4333022.
4.	1702.	9.	41007.	14.	7100010000.
5.	61040.	10.	700000.	15.	944312038[illegible].

ADDITION.

Mental Exercises.

236. 1. A boy gave 10 cents for a pencil, 5 cents for a piece of rubber, and 4 cents for a penholder. How much did he pay for all?

2. How many are 17 and 4? 27 and 4? 37 and 4? 67 and 4?

3. Begin with 5 and count by 5's to 65.

4. A man bought a cow for 35 dollars, a sheep for 8, and had 6 left. How many had he at first?

5. How many are 11, 7, and 9? 13, 4, and 3? 14, 5, and 8? 31, 6, and 7?

6. Jane had 19 cents and her father gave her 8. How many then had she?

7. In a garden are 6 cherry trees, 8 peach trees, and 12 pear trees. How many trees are there in the garden?

8. Count by 6's from 12 to 48. By 7's from 21 to 63.

9. How many are 9, 11, and 8? 14, 6, and 7? 23, 8, and 1? 32, 7, and 9?

10. James solved in the forenoon 13 problems, in the afternoon 8, and in the evening 6. How many did he solve in all?

11. Add by 3's from 1 to 37. By 9's from 9 to 81.

12. A man bought a harness for 31 dollars, a bridle for 8, and had 9 left. How many dollars had he at first?

13. How many are $13 + 6 + 4 + 9$?

14. Sold a cow for 35 dollars, a sheep for 7, and a calf for 6. How much did they all bring?

15. Mary had 17 apples, Jane gave her 11, and William 9. How many had she then?

16. How many are 23 and 47? 31 and 59?

17. A farmer has sheep in two folds; in one are 27 and in the other 35. How many are there in both folds?

Written Exercises.

237. What is the sum of

1. 163, 710, and 99?
2. 81, 309, and 777?
3. 361, 880, and 312?
4. 713, 554, and 946?
5. 316, 721, and 102?
6. 31, 605, 701, and 1305?
7. 203, 314, 445, and 676?
8. 3306, 92, 705, and 322?
9. 4008, 1982, 81, and 37?
10. 8117, 66, 314, and 705?
11. $5134 + 756 + 333 + 162 + 1451 =$ how many?
12. $93 + 71 + 562 + 459 + 19 + 6347 =$ how many?
13. $4456 + 340 + 781 + 62 + 17 + 9 + 13 =$ how many?

238. Add the numbers as they stand in columns, and also write those of the horizontal lines, in columns, and add.

	(14)	(15)	(16)	(17)	(18)
19.	341	1372	79304	31617	213
20.	420	8104	66772	7809	4576
21.	1562	16175	4076	421	319
22.	3412	40007	555	79	89
23.	4987	9312	719	5	74913

24. Write the numbers of Ex. 19 and Ex. 20, and add. Of Ex. 21, Ex. 22, and Ex. 23, and add.

SUBTRACTION.

Mental Exercises.

239. 1. How many is 7 less 2? 17 less 2? 27 less 2? 37 less 2? 47 less 2?

2. John had 11 cents and paid 5 cents for a pencil. How many had he left?

3. How many is 13 less 4? 23 less 4? 33 less 4? 43 less 4? 53 less 4?

4. Sold a knife for 31 cents, which was 6 cents more than it cost me. What did it cost me?

5. Count back from 30 to 0 by subtracting 3's? From 50 to 0 by subtracting 5's?

6. Ellen is 19 years old and her sister is 11. What is the difference in their ages?

7. Henry has 13 apples and Edmond 9. How many more has Henry than Edmond?

8. Count back by 6's from 72 to 6? From 96 to 72?

9. How many will remain if 9 be taken from 17? 11 from 20? 7 from 31?

10. Daniel has 8 cents, how many more cents does he need to make 19 cents?

11. If you have 6 peaches how many more must you have to make 15 peaches?

12. What number is 24 — 7? 35 — 6?

13. What number must be added to 16 to make 30?

14. Lucy had 14 apples and enough more were given her to make 27. How many were given her?

15. A boy had 38 cents and spent 19. How many had he then?

16. A farmer had 32 bushels of wheat and sold 15 bushels. How many had he left?

17. How much must be added to 43 to make 50?

Written Exercises.

240. Subtract the numbers as they stand in columns, and, also, as indicated by the sign: —

(1.) (2.)	(13.) (14.)	(25.) (26.)
3. 790 — 519	15. 597 — 314	27. 1604 — 912
4. 506 — 315	16. 109 — 19	28. 835 — 807
(5.) (6.)	(17.) (18.)	(29.) (30.)
7. 600 — 509	19. 719 — 685	31. 2800 — 1315
8. 297 — 220	20. 538 — 416	32. 1999 — 1207
(9.) (10.)	(21.) (22.)	(33.) (34.)
11. 902 — 516	23. 990 — 189	35. 67041 — 58908
12. 815 — 413	24. 468 — 91	36. 19052 — 11416

37. A merchant has property to the amount of 17675 dollars and owes 9679 dollars. How much will he have when his debts are paid?

MULTIPLICATION.

Mental Exercises.

241. 1. If a man can travel 6 miles in one hour, how far can he travel in 8 hours?

2. What will 9 melons cost, at 11 cents each?

3. How many are 3 times 7? 5 times 7?

4. How many are 2 times 6? 6 times 6?

5. If 4 men can build a fence in 9 days, how many men can build it in one day?

6. What is the product of 5 by 3? 5 by 8?

7. What will 12 barrels of flour cost, at 9 dollars each?

8. In a certain corn-field there are 12 rows, and 12 times as many hills in each row as there are rows. How many hills are there in each row?

9. Multiply 0 times 4 to 12 times 4.

10. Multiply 0 times 6 to 12 times 6.

11. A man can earn 8 dollars in a week. How much can he earn in 10 weeks?

12. If a quantity of provisions will last 7 men 11 weeks, how long will it last one man?

13. Multiply 0 times 5 to 12 times 5.

14. Multiply 0 times 7 to 12 times 7.

15. What will 11 tons of coal cost, at 8 dollars a ton?

16. How much will 12 barrels of flour cost, at 9 dollars a barrel?

17. How many is 8 times 6? 9 times 9? 10 times 11?

18. Multiply 0 times 8 to 12 times 8.

19. Multiply 0 times 9 to 12 times 9.

20. A man bought 10 sheep at 6 dollars each, and had 10 dollars left. How much had he at first?

21. Recite the table of 10's from 0 times 10 to 12 times 10

22. I had 12 marbles and Alfred gave me 5 times as many, less 8. How many had I then?

Written Exercises.

242. Multiply the numbers as they stand in columns, and also, as indicated by the sign.

(1.)	(2.)	(13.)	(14.)	(25.)	(26.)
3. 319	× 39	15. 517	× 121	27. 3121	× 314
4. 67	× 7	16. 71	× 17	28. 546	× 78
(5.)	(6.)	(17.)	(18.)	(29.)	(30.)
7. 826	× 43	19. 931	× 91	31. 4417	× 645
8. 48	× 8	20. 123	× 82	32. 231	× 50
(9.)	(10.)	(21.)	(22.)	(33.)	(34.)
11. 444	× 579	23. 719	× 53	35. 21461	× 209
12. 61	× 9	24. 47	× 74	36. 194	× 300

37. Multiply 319 by 7; 826 by 8; 444 by 9; 517 by 17, etc.

38. Multiply 39 by 67; 43 by 48; 579 by 61; 121 by 71, etc.

39. In a mile there are 5,280 feet; how many feet in 250 miles?

40. At the rate of 27 miles per hour, in how many hours will a train run 2,565 miles?

DIVISION.

Mental Exercises.

243. 1. How many oranges at 5 cents each can be bought for 40 cents?

2. How many pounds of prunes at 9 cents each can be bought for 54 cents?

3. At 8 dollars a ton, how many tons of coal can be bought for 56 dollars?

4. At 10 cents each, how many writing-books can be bought for 90 cents?

5. How many times 4 in 24? 7 in 28?

6. How many 6's in 42? 7's in 49?

7. At 3 cents each, how many lemons can be bought for 21 cents?

8. A factory has 64 windows; how many times 8 windows has it?

9. Divide by 3 from 3 in 9 to 3 in 36.

10. Divide by 5 from 5 in 10 to 5 in 60.

11. Seven days are 1 week; what part of a week is 1 day?

12. What part of 10 dollars are 2 dollars?

13. What is one-sixth of 72?

14. James divided 32 apples equally among his 4 brothers. How many did each of his brothers receive?

15. Divide by 4 from 4 in 4 to 4 in 48.

16. Divide by 6 from 6 in 18 to 6 in 72.

17. What part of 16 is 4? Of 18 is 6?

18. How many 8's in 64? 9's in 72?

19. If 90 cents be divided among 10 boys, how many cents will each boy receive?

20. A man paid 81 dollars for 9 barrels of flour; how much did he pay a barrel?

21. Divide by 7 from 7 in 14 to 7 in 84.

22. Divide by 9 from 9 in 9 to 9 in 108.

23. What is the value of $14 \div 2$? $\frac{16}{2}$? $\frac{18}{6}$?

24. How many sacks holding 8 quarts each can be filled from 75 quarts, and how many quarts will remain?

25. In 103 eggs are how many dozen, and how many remain?

26. What is one tenth of 80? One twelfth of 108?

27. Divide by 10 from 10 in 10 to 10 in 120.

28. Divide by 12 from 12 in 36 to 12 in 144.

29. I have 150 dollars; how many coats at 12 dollars each can I buy with the money, and how many dollars will remain?

Written Exercises.

244. Divide

1. 3160 by 5.	9. 673 by 13.	17. 3140 by 310
2. 1648 by 4.	10. 1631 by 10.	18. 11472 by 331
3. 2739 by 3.	11. 9785 by 31.	19. 12455 by 19
4. 5047 by 7.	12. 900 by 100.	20. 47839 by 46
5. 8438 by 9.	13. 1473 by 17.	21. 14000 by 75
6. 4416 by 8.	14. 3755 by 15.	22. 914131 by 121
7. 3550 by 2.	15. 836 by 16.	23. 31441 by 1000
8. 7332 by 6.	16. 2025 by 25.	24. 79346 by 2003

25. Divide 673 by 5; 1731 by 4; 9785 by 3; 900 by 7, etc.

26. Divide 3140 by 13; 11472 by 10; 12455 by 31; etc.

27. At 85 dollars per acre how many acres of land can be bought for 8,751 dollars, and how many dollars will remain?

UNITED STATES MONEY.

Mental Exercises.

245. 1. In \$3 how many dimes?

2. I have \$1.50 and wish to divide it among 3 boys; how many dimes can I give each?

3. John has 5 cents, Edwin 8, Henry 3, and George 4; how many have they all?

4. What will 8 quarts of milk cost at 7 cents a quart?

5. At 4 cents each how many oranges can I buy for 30 cents, and how many cents will remain?

6. Bought 4 apples at 2 cents each, and 3 pears at 3 cents each. How much did they all cost?

7. Sarah has 10 cents, and Jane 3 more than Sarah. How many have both?

8. In a box containing 20 cents, John put 14 cents, Mary 5, and Lucy 8. How many cents were there then in the box?

9. John had 21 cents; he lost 4 cents and his mother gave him 5; how many had he then?

10. A man paid $72 for a watch, and one-eighth as much for a chain; what was the cost of both?

11. If apples cost 2 cents apiece and peaches 4 cents apiece, what is the cost of 4 apples and 10 peaches?

12. A boy who had 50 cents paid 10 cents for marbles, 8 cents for a top, lost 3 cents, and bought 8 oranges at 3 cents apiece. How many cents did he have left?

13. How many dollars in 450 cents? In 500 cents?

14. How are dollars and cents in the same expression separated or distinguished?

15. A man having $47 buys 6 cords of wood at $7 per cord; how much money has he left?

16. Mills are reduced to dollars by pointing off three orders of figures. Why?

17. Bought 12 yards of cloth for $1.44, what was the cost per yard?

18. In what way may a bill be made to show that it has been paid?

Written Exercises.

246. 1. Write and add: five dollars four cents, eighteen dollars eighty cents, and fifty-three dollars.

2. Gave for a horse $250, for a wagon $150.50; and for a harness $27.87½. How much was given for the whole?

3. Johnson is worth $31,675 and Henshaw $1,975.50. How much more is Johnson worth than Henshaw?

4. How much must be paid for 37 yards of ribbon at 19 cents a yard?

5. Paid $5.27 for 17 boxes of berries; what was that a box?

6. How much must be paid for 115 tons of coal at $8.75 per ton?

7. A man bought a horse for $150, and 2 yoke of oxen at $175.50 each. How much did the whole cost him?

8. Bought a box of raisins for $3.15, a tub of butter for $14.50, and flour for $53.65. The whole was sold at a profit of $9.65; what sum did the whole bring?

9. A father died leaving $12,560, which was divided equally among 8 heirs. How much was each heir's share?

10. Bought a farm of 110 acres for $6,875. How much was it per acre?

11. Bought a farm containing 40 acres of meadow and 17 woodland, for $2,850. Sold to one man 10 acres of woodland for $85 per acre; to another a house lot of one acre for $90, and the remainder to a third for $2,025. What did I gain by the operation?

12. What is the value of 5,840 pounds of butter, at 35 cents per pound?

13. At $8 per pair how many pairs of boots can I buy for $29,008?

14. New Haven, May 16, 1876. William Peck owed Samuel Dean; for 5 pounds of tea, at 75 cents a pound, bought March 11; 18 pounds of granulated sugar, at 12 cents a pound, bought March 31; and 6 gallons of molasses, at 76 cents a gallon, bought April 20. Make out the bill in proper form and find the amount due.

15. Worcester, June 17, 1876. William Thomas bought of Albert Smith, 16 Practical Arithmetics, at 62 cents each, 15 Elementary Arithmetics, at 50 cents each, 5 Dictionaries, at $1.25 each, and 1 dozen Fifth Readers, at $7.50. Make out the bill in proper form, find the amount, and receipt it for Albert Smith.

REDUCTION OF FRACTIONS.

Mental Exercises.

247. 1. If an apple be divided in two equal parts, and each of these parts be divided into two equal parts, what part of the apple will be one of the pieces?

2. *One half is* how many fourths?

3. If an apple be divided into two equal parts, and each of these parts be divided into three equal parts, what part of the apple will one of these pieces be?

4. One half is how many sixths?

5. The fourths of a number are how many times the halves? The sixths of a number are how many times the halves?

6. The eighths of a number are how many times the fourths? The eighths of a number are how many times the halves?

7. The sixths of a number are how many times the thirds? The ninths of a number are how many times the thirds?

8. How many halves are equal to $\frac{2}{4}$? To $\frac{3}{6}$? To $\frac{4}{8}$?

9. Express $\frac{5}{10}$ in lower terms, $\frac{9}{12}$ in fourths.

10. In 1 apple there are how many halves of 1 apple? In 3 apples?

11. In 1 orange there are how many sixths of 1 orange? In 8 oranges?

12. How many halves of one apple in $3\frac{1}{2}$ apples? How many thirds of an orange in $4\frac{1}{3}$ oranges?

13. Change $3\frac{1}{2}$ to halves. $4\frac{1}{3}$ to thirds.

14. How many dollars in 2 half dollars? In $\frac{7}{2}$ dollars? In $\frac{13}{3}$ dollars?

15. Jane has $\frac{1}{3}$ of an apple and Arthur $\frac{8}{12}$. How many sixths of an apple have each?

16. Change $\frac{2}{3}$ and $\frac{3}{5}$ to fifteenths? $\frac{3}{4}$ and $\frac{6}{8}$ to fourths? $\frac{2}{7}$ and $\frac{1}{2}$ to fourteenths?

Written Exercises.

248. Reduce to their lowest terms:—

1. $\frac{12}{15}$.	4. $\frac{56}{63}$.	7. $\frac{54}{75}$.
2. $\frac{27}{36}$.	5. $\frac{108}{120}$.	8. $\frac{123}{144}$.
3. $\frac{66}{100}$.	6. $\frac{224}{672}$.	9. $\frac{324}{396}$.

10. Reduce 17 to thirds.

11. Reduce 68 to a fraction whose denominator shall be 15.

12. Reduce $81\frac{1}{7}$ to an improper fraction.

13. Reduce $19\frac{7}{20}$ to an improper fraction.

14. Reduce $\frac{87}{11}$ to a whole or mixed number.

15. Express the value of $\frac{1509}{18}$ in an integer.

16. Express the value of $\frac{5982}{72}$ in a mixed number.

17. Reduce $\frac{2}{5}$ and $\frac{5}{8}$ to fortieths.

18. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ to a common denominator.

19. Reduce $\frac{4}{5}$ and $\frac{6}{7}$ to a common denominator.

20. Reduce $\frac{1}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$ to a common denominator.

ADDITION OF FRACTIONS.

MENTAL EXERCISES.

249. 1. James gave $\frac{1}{4}$ of an apple to John, and $\frac{1}{2}$ of an apple to Susan. What part of an apple did he give away?

2. How many fourths are $\frac{1}{4}$ and $\frac{1}{2}$?

3. If Henry has $\frac{3}{5}$ of an apple and Arthur $\frac{4}{5}$, how many fifths have both?

4. How many eighths are $\frac{3}{4}$ and $\frac{5}{8}$?

5. If you should give $\frac{1}{2}$ of a dollar for a book, and $\frac{1}{3}$ of a dollar for a slate, what part of a dollar would you spend?

6. If you should give your brother $\frac{3}{8}$ of a melon, your sister $\frac{1}{8}$, and a playmate $\frac{2}{8}$, what part of the melon would you give away?

7. How many eighths are $\frac{1}{8} + \frac{2}{8} + \frac{3}{8}$?

8. How many are $\frac{5}{6} + \frac{2}{6}$? $\frac{4}{7} + \frac{3}{7}$?

9. How many are $2\frac{1}{2} + 3\frac{1}{4}$? $1\frac{2}{3} + 2\frac{1}{6}$?

10. A farmer has in one garden $\frac{5}{16}$ of an acre, and in another $1\frac{5}{8}$ acres. What is the extent of the two gardens?

WRITTEN EXERCISES.

250. Add as indicated by the sign, —

1. $\frac{3}{4} + \frac{6}{2} + \frac{7}{12}$.	4. $\frac{13}{8} + \frac{5}{16}$.	7. $5\frac{1}{2} + 4\frac{5}{6}$.
2. $\frac{5}{6} + \frac{1}{3} + \frac{5}{6}$.	5. $\frac{4}{9} + \frac{5}{6}$.	8. $\frac{1}{2} + \frac{2}{3} + 1\frac{1}{4}$.
3. $\frac{3}{5} + \frac{9}{10} + \frac{3}{4}$.	6. $\frac{11}{8} + 1\frac{1}{2}$.	9. $1\frac{1}{2} + \frac{2}{5} + \frac{1}{3}$.

10. What is the sum of $13\frac{1}{15}$, $\frac{1}{15}$, and $\frac{2}{5}$?

11. What is the sum of $\frac{7}{8}$, $\frac{6}{20}$, and $3\frac{1}{10}$?

12. What is the sum of $4\frac{3}{4}$, 5, and $6\frac{1}{5}$?

SUBTRACTION OF FRACTIONS.

Mental Exercises.

251. 1. David had $\frac{3}{4}$ of an apple, and gave his brother $\frac{1}{4}$ of an apple. What part of an apple had he left?

2. How much is $\frac{3}{4}$ less $\frac{2}{4}$? $\frac{3}{4}$ less $\frac{1}{2}$?

3. If you should have $\frac{7}{8}$ of a cake and give away $\frac{5}{8}$, what part of the cake would you have left?

4. How much is $\frac{7}{8}$ less $\frac{5}{8}$? $\frac{15}{16}$ less $\frac{9}{16}$?

5. If you should have \$1 and should give away $\frac{3}{8}$ of a dollar, what part of a dollar would you have left?

6. If a man owning $\frac{7}{8}$ of a shop should sell $\frac{1}{2}$ of the shop, what part of it would he then have?

7. How much is $\frac{7}{8}$ less $\frac{1}{2}$? 1 less $\frac{3}{8}$?

8. How much must be added to $\frac{3}{8}$ to make $1\frac{1}{4}$?

9. How many are $3\frac{1}{3}$ less $\frac{5}{6}$? $4\frac{1}{5}$ less $\frac{3}{4}$?

10. Bought 2 tons of coal and sold $1\frac{5}{8}$ tons. How much was then left?

Written Exercises.

252. Subtract as indicated by the sign: —

1. $\frac{11}{12} - \frac{3}{4}$.	4. $\frac{15}{16} - \frac{7}{8}$.	7. $4\frac{1}{16} - 3\frac{1}{2}$.
2. $\frac{17}{20} - \frac{3}{4}$.	5. $\frac{11}{5} - \frac{9}{10}$.	8. $4\frac{6}{24} - 3\frac{4}{8}$.
3. $\frac{13}{16} - \frac{1}{8}$.	6. $\frac{71}{30} - \frac{21}{15}$.	9. $6\frac{7}{14} - \frac{1}{7}$.

10. What is the difference between $4\frac{5}{6}$ and $2\frac{1}{12}$?

11. How much must be added to $5\frac{5}{6}$ to make 19?

12. How much greater is the sum than the difference of $3\frac{1}{3}$ and $2\frac{5}{6}$?

MULTIPLICATION OF FRACTIONS.

Mental Exercises.

253. 1. What part of a melon is 5 times $\frac{3}{16}$?

2. An owner of a ship sold to each of 4 men $\frac{7}{32}$ of it. What part of the ship did he sell?

3. How much is 5 times $\frac{8}{16}$? 4 times $\frac{7}{32}$?

4. If a boy can gather $\frac{7}{24}$ of a bushel of berries in 1 day, how many can he gather in 6 days?

5. A man can earn $\$2\frac{1}{2}$ in 1 day; how much can he earn in 7 days?

6. How much is 6 times $\frac{7}{24}$? 7 times $2\frac{1}{2}$?

7. How much is 8 times $\frac{8}{64}$? 9 times $\frac{11}{108}$?

8 How much is 10 times $3\frac{3}{4}$? 11 times $2\frac{3}{7}$?

9. What part of 1 apple is $\frac{1}{5}$ of 2 apples?

10. What is $\frac{1}{4}$ of 3 apples? $\frac{1}{6}$ of 5 oranges?

11. What is $\frac{1}{4}$ of 12? $\frac{1}{6}$ of 15? $\frac{1}{7}$ of 14?

12. What is $\frac{2}{4}$ of 12? $\frac{2}{5}$ of 15? $\frac{2}{7}$ of 14?

13. If you can do a piece of work in 16 days, in what time can you do $\frac{3}{4}$ of it?

14. What is $\frac{5}{7}$ of 35? $\frac{7}{11}$ of 66? $11\frac{1}{2}$ of 84?

15. If a cord of wood be worth \$8, how much will $5\frac{3}{4}$ cords be worth?

16. How much is $5\frac{3}{4}$ times 8? $7\frac{2}{3}$ times 12?

17. If $\frac{1}{2}$ of an apple be cut into two equal parts, what part of the apple will one of the parts be?

18. If $\frac{1}{3}$ of an orange be cut into two equal parts, what part of the orange will one of the parts be?

19. What is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$?

20. If a boy having $\frac{3}{4}$ of a melon should separate it into two equal parts, what part of the melon will one of the parts be?

21. What is $\frac{1}{2}$ of $\frac{3}{4}$? $\frac{1}{3}$ of $\frac{2}{5}$? $\frac{1}{4}$ of $\frac{5}{6}$?

22. A boy having $\frac{3}{4}$ of a bushel of nuts, sold $\frac{2}{3}$ of the quantity; what part of a bushel did he sell?

23. A merchant owning $\frac{5}{12}$ of a ship sells $\frac{2}{5}$ of that part; what part of the ship did he sell?

24. What is $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{3}{5}$ of $\frac{5}{12}$? $\frac{3}{4}$ of $\frac{7}{8}$?

25. What will $2\frac{1}{4}$ bushels of potatoes cost, at $\frac{3}{4}$ of a dollar a *bushel?*

26. What will $6\frac{1}{2}$ yards of cloth cost, at $\frac{8}{10}$ of a dollar a yard?

27. What is $\frac{3}{4}$ of $2\frac{1}{4}$? $\frac{3}{10}$ of $6\frac{1}{2}$? $\frac{2}{5}$ of $8\frac{2}{5}$?

Written Exercises.

254. Multiply as indicated by the sign: —

1. $\frac{5}{8} \times 12$.	4. $16 \times \frac{3}{4}$.	7. $\frac{5}{6} \times \frac{3}{5}$.
2. $\frac{4}{5} \times 7$.	5. $17 \times \frac{2}{3}$.	8. $\frac{11}{16} \times \frac{4}{5}$.
3. $4\frac{1}{2} \times 8$.	6. $10 \times 3\frac{1}{2}$.	9. $3\frac{1}{10} \times 2\frac{2}{3}$.

10. At $\frac{7}{8}$ of a dollar a yard, what must be paid for 56 yards of cloth?

11. A certain estate is worth \$14,000; what is the value of $\frac{11}{35}$ of it?

12. Johnson owns $\frac{27}{32}$ of a ship, and his brother owns $\frac{5}{9}$ as much. What part of the ship does his brother own?

13. Multiply $161\frac{5}{12}$ by $31\frac{4}{5}$. $890\frac{7}{8}$ by $\frac{15}{16}$.

14. What cost $11\frac{3}{5}$ tons of coal at $\$7\frac{5}{8}$ per ton?

15. What will $16\frac{1}{2}$ quarts of strawberries come to at $26\frac{1}{4}$ cents per quart?

DIVISION OF FRACTIONS.

Mental Exercises.

255. 1. If $\frac{2}{5}$ of an apple be divided between two boys, what part of the apple will each receive?

2. Jane had $\frac{6}{7}$ of a melon, and divided the same among 3 playmates; what part of the melon did each receive?

3. What part of 1 is $\frac{1}{2}$ of $\frac{2}{5}$? $\frac{1}{3}$ of $\frac{6}{7}$?

4. How much is $\frac{9}{14}$ divided by 3? $\frac{8}{15}$ divided by 9?

5. If 5 boys have $\frac{7}{8}$ of a melon divided among them, what part of the melon will each receive?

6. What part of 1 is $\frac{1}{6}$ of $\frac{7}{8}$? $\frac{1}{4}$ of $\frac{5}{6}$?

7. If 1 man can do a piece of work in $\frac{9}{10}$ of a day, in what time can 6 men do it?

8. If 5 bushels of apples cost $\$6\frac{1}{4}$, what is the cost of 1 bushel?

9. How many fourths of an orange in 1 orange? In 2 oranges?

10. In \$4 how many times $\frac{3}{4}$ of a dollar?

11. How many books at $\frac{2}{5}$ of a dollar each can be bought for \$10?

12. How much is $4 \div \frac{3}{4}$? $10 \div \frac{3}{5}$?

13. How much is $11 \div \frac{2}{3}$? $12 \div \frac{7}{8}$?

14. At $2\frac{1}{2}$ cents a pound, how many pounds of chalk can be bought for 20 cents?

15. If cloth is $\frac{1}{4}$ of a dollar a yard, how many yards can be bought for $\frac{3}{4}$ of a dollar?

16. At $\frac{1}{5}$ of a dollar a quart, how many quarts of strawberries can be bought for $\frac{4}{5}$ of a dollar?

17. How many times is $\frac{1}{4}$ contained in $\frac{3}{4}$? $\frac{1}{5}$ in $\frac{4}{5}$?

18. At $\frac{2}{5}$ of a dollar a gallon, how many gallons of vinegar can be bought for $\frac{4}{5}$ of a dollar?

19. How many times $\frac{2}{5}$ in $\frac{4}{5}$? $\frac{2}{15}$ in $\frac{14}{15}$?

20. At $\frac{3}{10}$ of a dollar each, how many slates can be bought for $\frac{3}{5}$ of a dollar?

21. How many times $\frac{3}{10}$ in $\frac{3}{5}$? $\frac{5}{16}$ in $\frac{3}{4}$?

Written Exercises.

256. Divide as indicated by the sign:—

1. $\frac{28}{29} \div 7$.	4. $12 \div \frac{3}{4}$.	7. $\frac{7}{8} \div \frac{2}{3}$.
2. $\frac{18}{19} \div 4$.	5. $18 \div \frac{5}{7}$.	8. $1\frac{1}{12} \div \frac{4}{5}$.
3. $\frac{6}{7} \div 4$.	6. $16 \div 2\frac{1}{4}$.	9. $6\frac{1}{5} \div 2\frac{1}{10}$.

10. How much is $16\frac{1}{4}$ divided by 5?

11. How many tons of coal at $\$7\frac{3}{4}$ a ton can be bought for \$31?

12. When oranges are $6\frac{1}{4}$ cents each, how many can be bought for $87\frac{1}{2}$ cents?

DECIMALS.

MENTAL EXERCISES.

257. 1. If an apple be divided into 10 equal parts, what will 1 of the parts be called? 2 of the parts?

2. How much is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{100}$? $\frac{1}{10}$ of $\frac{1}{1000}$?

3. What part of $\frac{1}{10}$ is $\frac{1}{100}$? What part of $\frac{1}{100}$ is $\frac{1}{1000}$?

4. When the decimal is a part of a mixed number how may we read the integral part?

5. How many hundredths are 2 tenths?

6. How many thousandths are 3 tenths?

7. Three hundredths are equivalent to what common fraction?

8. One fifth is equal to how many tenths?

9. One fourth is equal to how many hundredths?

10. Three fourths are equal to how many hundredths?

11. If you pay 5 hundredths of a dollar for a pencil and 25 hundredths of a dollar for a slate, what part of a dollar do you pay for both?

WRITTEN EXERCISES.

258. 1. Express .7 as hundredths? Express as having a common denominator .7 and .05?

2. Reduce .625 to a common fraction.

3. Change $\frac{5}{8}$ to an equivalent decimal.

4. Change .029 to a common fraction.

5. Reduce $\frac{5}{17}$ to a decimal of four places.

259. What is the value of

1. $3.14 + .005 + .619$.	6. $7.04 \times .20$.	11. $525 \div .25$.
2. $.06 + 17.5 + 95$.	7. $4.5 \times .06$.	12. $410 \div .05$.
3. $63.51 - 9.005$.	8. $144 \times .24$.	13. $1.08 \div 1.2$.
4. $8.1003 - .0956$.	9. $.08 \times .005$.	14. $1.47 \div .00147$.
5. $19 - .00001$.	10. 11.05×2.06.	15. $.01728 \div .012$.

COMPOUND NUMBERS.

Mental Exercises.

260. 1. John gathered 3 pecks of nuts. How many gills would they make?

2. Susan bought at one time 8 ounces of candy, at another 1 pound 2 ounces, and at another 6 ounces. How much did she buy in all?

3. From a 5 gallon keg of molasses there has been drawn out 2 gallons 3 quarts. How much remains in the keg?

4. James has three lines each 2 yards 2 feet in length. What is the entire length of the three?

5. A farmer divided 5 pounds 4 ounces of maple sugar equally among 4 boys. How much did each boy receive?

Written Exercises.

261. Reduce

13. T. 7 cwt. 10 lb. to pounds.
2. 10 oz. 13 pwt. 8 gr. to grs.
3. 15 rd. 3 yd. 2 ft. to feet.
4. 5 A. 120 sq. rd. to sq. feet.
5. 15 cu. yd. 21 cu. ft. to cu. ft.
6. 16 bu. 3 pk. 3 qt. to pints.
7. 2 hhd. 32 gal. 1 qt. to quarts.
8. 2 y. 300 da. 6 h. to hours.
9. 90° 30′ 30″ to seconds.
10. 501 quarts to bushels.
11. 445577 feet to miles.
12. 947 sq. rd. to acres.
13. 224 feet to rods.
14. 17424 sq. ft. to square rods.
15. 730960 cu. ft. to cubic yds.
16. 568240 seconds to hours.
17. 6048 gills to hogsheads.
18. 32610 seconds to degrees.

19. Add 1 lb. 6 oz. 15 pwt., 5 lb. 0 oz. 13 pwt., and 10 oz. 16 pwt.
20. Add 3 mi. 120 rd. 5 yd. 1 ft., and 15 mi. 160 rd. 3 yd. 2 ft.
21. Subtract 1 mi. 35 rd. 4 yd. 2 ft. from 5 mi. 30 rd. 3 yd. 1 ft.
22. Subtract 9 cwt. 45 lb. 9 oz. from 1 T. 8 cwt. 43 lb. 8 oz.
23. Multiply 16 bu. 3 pk. 4 qt. by 6; by 11.
24. Multiply 4 A. 110 sq. rd. 10 sq. yd. by 7; by 12.
25. Divide 9 bu. 1 pk. 3 qt. by 2; 37 A. 56 P. by 8.
26. *Divide 10 hhd. 39* gal. 2 qt. by 7; 79 cwt. 44 lb. by 12.

PERCENTAGE.

Mental Exercises.

262. 1. A farmer had 200 chickens, and the hawks took 5 per cent. of them. How many did the hawks take?

2. What per cent. of 100 is 5? Of 200 is 5?

3. When from a school of 50 pupils 6 per cent. of the number stay at home, how many are absent from school?

4. What per cent. of 50 is 3? Of 100 is 6?

5. A barrel of apples had $\frac{1}{4}$ part unsound. What per cent. of the whole was unsound?

6. From a car load of 10 tons of coal, 2 tons have been sold, what per cent. has been sold?

7. An agent sold goods to the amount of $300. What was his commission at $2\frac{1}{2}$ per cent.?

8. From a cistern of water containing 150 gallons there was a loss of 10 per cent. from leakage. How much was the loss?

9. When a barrel of flour which cost $10 is sold for $11, what is the gain per cent?

10. A horse which cost $120 is sold for $90. What is the loss per cent?

11. When sugar which cost 10 cents is sold for 12 cents, what is the gain per cent?

12. When an article which cost $25 is sold for $30, what is the gain per cent?

13. I had $300, but paid away 10 per cent. of it. How many dollars had I then left?

14. I have 80 cents; how much of it must I spend to have 75 per cent. left?

15. A merchant sold tea at a profit of 15 cents per pound, which was 25 per cent. of the cost. What was the cost?

16. What per cent. of anything is $\frac{1}{8}$th part of it?

17. If I sell a watch for $60, which cost me $75, what per cent. is my loss?

Written Exercises.

263. What is

1. 4 % of 125 bushels?
2. $2\frac{1}{2}$ % of $6\frac{1}{3}$ miles?
3. 2 % of 350 yards?
4. $12\frac{1}{2}$ % of \$1400?
5. 25 % of 150 acres?
6. 40 % of 1275 tons?

What per cent. of

7. \$200 is \$40?
8. 863 tons is 34.52 tons?
9. \$6500 is \$975?
10. 225 yards is 9 yards?
11. 2000 feet is 5 feet?
12. 1000 lbs. is 125 lbs.?

INTEREST.

Mental Exercises.

264. 1. When 6 per cent. is allowed for the use of 100 cents, or \$1.00, what is the sum allowed?

2. What is the interest of \$100 for 1 year at 6 per cent? At 7 per cent.?

3. What is the interest of \$200 for 1 year at 7 per cent.? At 8 per cent.?

4. What is the amount of \$225 for 1 year at 4 per cent.? At 12 per cent.?

5. What is the interest of \$320 for 1 year at 5 per cent.? At 10 per cent.?

6. What is the interest of \$100 for 6 months, or for $\frac{1}{2}$ of a year at 6 per cent.?

7. What is the interest of 100 cents for 4 months at 9 per cent.? At 12 per cent.?

Written Exercises.

265. What is the interest of

1. \$900 at 6 % for 8 months.
2. \$250 at 5 % for 6 months.
3. \$75 at 8 % for 30 days.
4. \$108 at 7 % for 60 days.
5. \$50 at $6\frac{1}{2}$ % for 2 years.
6. \$90 at 10 % for $3\frac{1}{2}$ years.
7. \$6.50 at 7 % for 5 years.
8. \$11.80 at 6 % for 8 years.
9. What is the interest of \$500 for 1 year 6 months 15 days *at 6 per cent.*?

TEST EXAMPLES.

266. The **Test Examples** here given are designed for use in examining the proficiency of classes in the portions of the book to which the examples relate.

The articles in parenthesis denote parts of the text.

(ART. 1-66.)

1. Texas contains 237,321 square miles; express that number in words.

2. In 1870, there were in the United States five million six hundred forty-three thousand five hundred thirty-four illiterate persons; express that number in figures.

3. Add one million thirty three thousand seven hundred five and thirty-three million one thousand fifty-seven.

4. The sum of two numbers is 316,452, and one of the numbers is 209,753. What is the other number?

5. Show by an example how subtraction is proved by addition.

6. How much must be paid for a farm of 445 acres at 105 dollars per acre?

7. What is the value of $(3604+162)\times 3$?

8. How many farms of 160 acres are contained in 49760 acres?

9. Show by an example how multiplication and division prove each other.

10. A man having 16450 dollars, gives his son 5500 dollars, and divides the remainder among his three daughters. How much does each daughter receive?

(ART. 66-94.)

1. Express in figures, seventy-nine dollars seven cents five mills.

2. Smith's farm is worth thirteen thousand four hundred sixty dollars, his cattle six hundred forty-three dollars fifty cents, and his farming tools two hundred three dollars eighty-eight cents. How much is the whole worth?

3. Jones is worth four thousand twenty-five dollars, and Ford three thousand nine hundred dollars eighty-seven cents. How much is Jones worth more than Ford?

4. Bought a bill of goods, and on paying gave a five hundred dollar note. If the change paid me was $169.73 what was the amount of the bill?

5. How much must be paid for 3261 bushels of wheat, at $2.13 per bushel?

6. A drover has $5780, and wishes to purchase as many cows with it as he can at $80 each, and with the balance of the money sheep at $4 each. How many cows and sheep can he purchase?

7. What is the value of ($446 — $132) ÷ 17.

8. Of what prime factors is 1110 the product?

9. How many yards of cloth, at $3 per yard, must be given for 20 tons of coal, at $7.50 per ton?

10. At $7.50 per ton, how many tons of coal must be given for 1250 yards of cloth, at $3 per yard?

11. If 18 men can do a piece of work in 8 days, in what time can 6 men do it?

12. How many bushels of apples, at 32 cents a peck, *must be given* for 4 bushels of peaches, at 40 cents a *peck?*

(ART. 95-150.)

1. Express by figures one hundred thirteen three hundredths.

2. Express decimally by figures fifty and three hundred five ten-thousandths.

3. Show by an example what is meant by the terms of a fraction.

4. A man owned .25 of a mill, and afterwards bought at one time $\frac{3}{8}$, and at another $\frac{3}{16}$; how much of the mill did he then own?

5. If a man can build $12\frac{2}{5}$ rods of wall in one week, how many rods can he build in 52 weeks?

6. How many bushels of oats, at $\frac{5}{8}$ of a dollar a bushel, can be bought for $100?

7. Show by an example what is the denominator of a decimal fraction.

8. What common fraction is equivalent to .36?

9. What is the value of .7056 divided by .84, and .12 plus .9?

10. What decimal is equivalent to $\frac{1}{12}\frac{9}{5}$?

11. A owes $1000 to each of four persons; to one he pays $\frac{1}{2}$ of his debt, to another .75, to another $\frac{4}{5}$, and to the fourth .95. What is he then owing to the four?

(ART. 151-194.)

1. A boy sold 3 bu. 3 pk. 4 qt. of chestnuts at 6 cents a quart; how much did he get for them?

2. Bought 5 cwt. 20 lb. 8 oz. of opium at $7.75 per pound; how much did it cost me?

3. Show by an example that reduction descending and reduction ascending are the proof of each other.

4. How old, on April 12, 1873, was a man who was born May 16, 1819?

5. My farm contains 340 A. 2 R. 24 sq. rd.; how many acres has a farm which is $\frac{3}{4}$ of its size?

6. Calling a year 365 d. 5 h. 48 m. 50 sec., how many seconds are there in 6 years?

7. A man walked one day 15 m. 160 rd., the second day 21 m. 185 rd., and the third day 16 m. 131 rd. What was the average per day?

(ART. 195-220.)

1. Express decimally by figures 125 per cent.

2. What per cent. of 51 tons are 8.50 tons?

3. A man sold some property which cost him $2560, at a profit of 12½ %, how much did he get for it?

4. Of what is 2.80 yards ⅝ %?

5. Show by an example that a per cent. of a number is the same number of hundredths of that number.

6. Bought a horse for $93 and sold him for $99.51, what was the gain per cent.?

7. Sold a carriage, which cost me $350, for $320.25, what was the loss per cent.?

8. What is the interest of $500.50 for 1 y. 6 mo. 15 d. at 6 %?

9. Taking out the bank discount on a note for $4000 on 60 days at 7 %, how much of its face will remain?

10. What rate per cent. of profit will be realized from *an 8 per cent. stock* bought at 125?

Greenleaf, Benjamin
New elementary ari